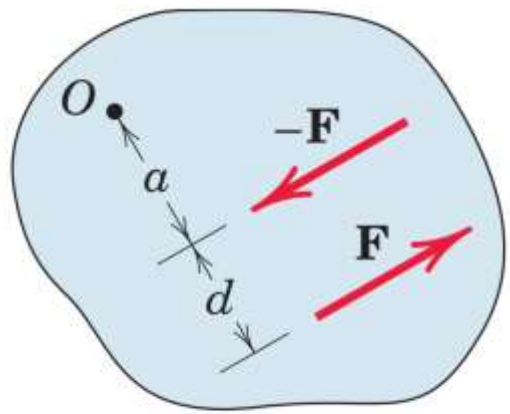
Lecture 5
Engineering Mechanics

Couple and Couple Moment

- The moment produced by two equal, opposite, and non-collinear forces is called a *couple*.



- M=F(a+d)-Fa
- M=Fd

- Its direction is counterclockwise

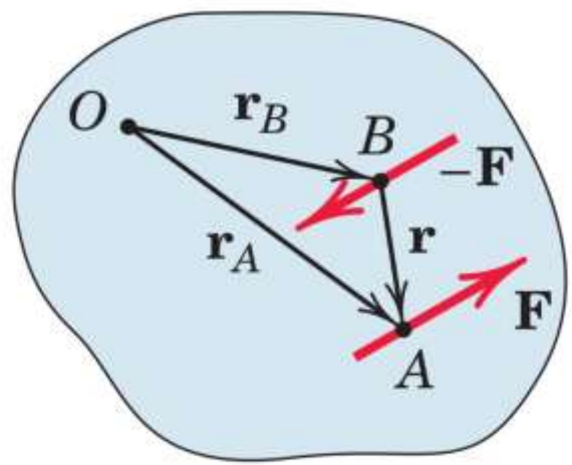
Vector

- We may also express the moment of a couple by using vector algebra.

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) = \vec{r}_A \times \vec{F} - \vec{r}_B \times \vec{F}$$

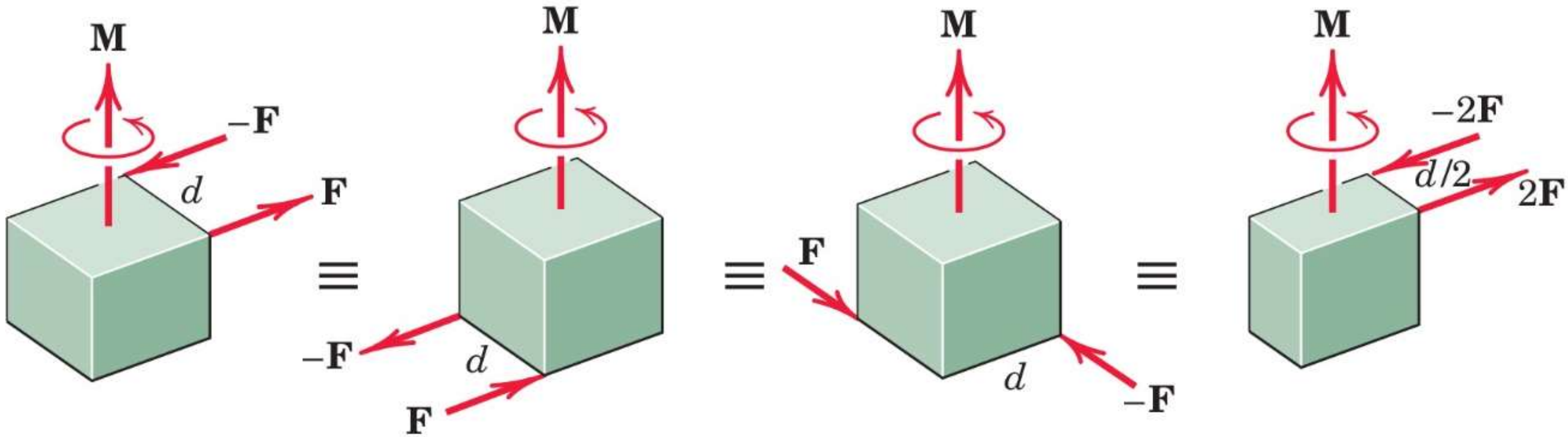
$$= (\vec{r}_A - \vec{r}_B) \times \vec{F} = \vec{r} \times \vec{F}$$

$$\Rightarrow \vec{M} = \vec{r} \times \vec{F}$$



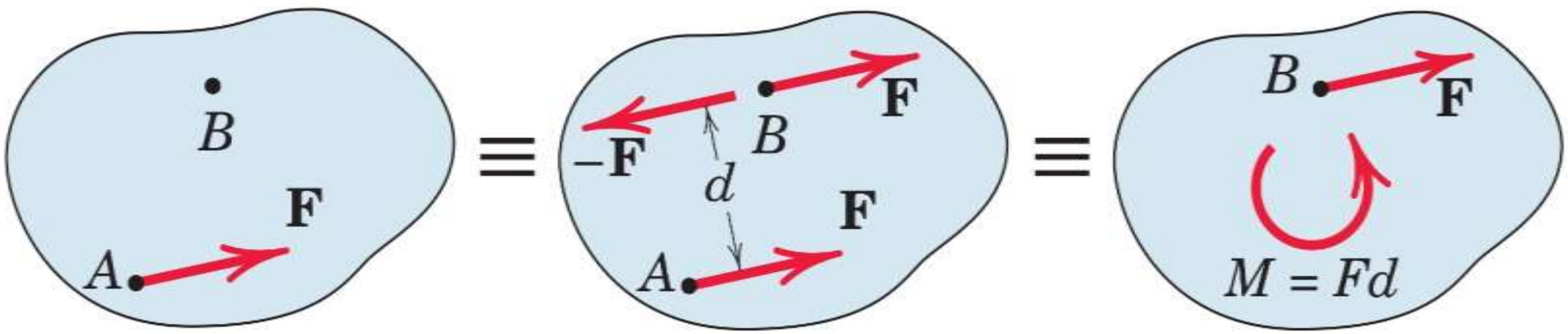
- where $\mathbf{r}_A$ and $\mathbf{r}_B$ are position vectors which run from point O to arbitrary points A and B on the lines of action of **F** and **F**, respectively.

Equivalent Couples



The Figure shows four different configurations of the same couple M.

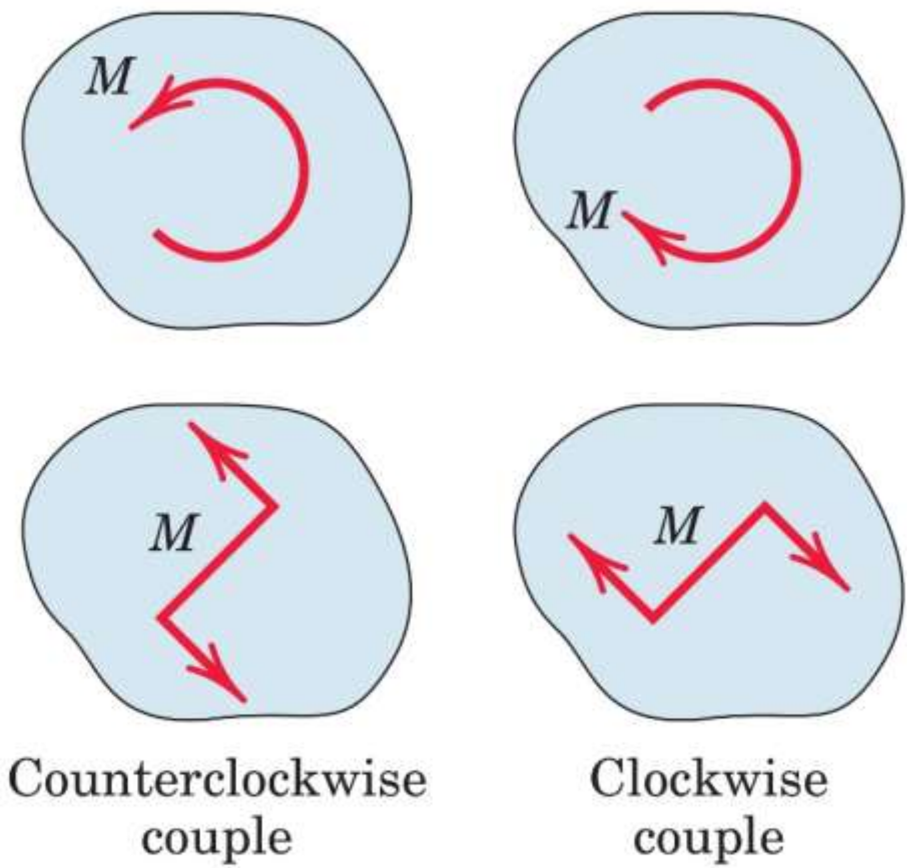
Force-Couple Systems



The replacement of a force by a force and a couple is illustrated in the Figure, where the given force F acting at point A is replaced by an equal force F at some point B and the counterclockwise couple M=Fd. The transfer is seen in the middle figure, where the equal and opposite forces F and F are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple M=Fd, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of the

Figure is referred to as a force–couple system.

Counterclockwise and Clockwise couples

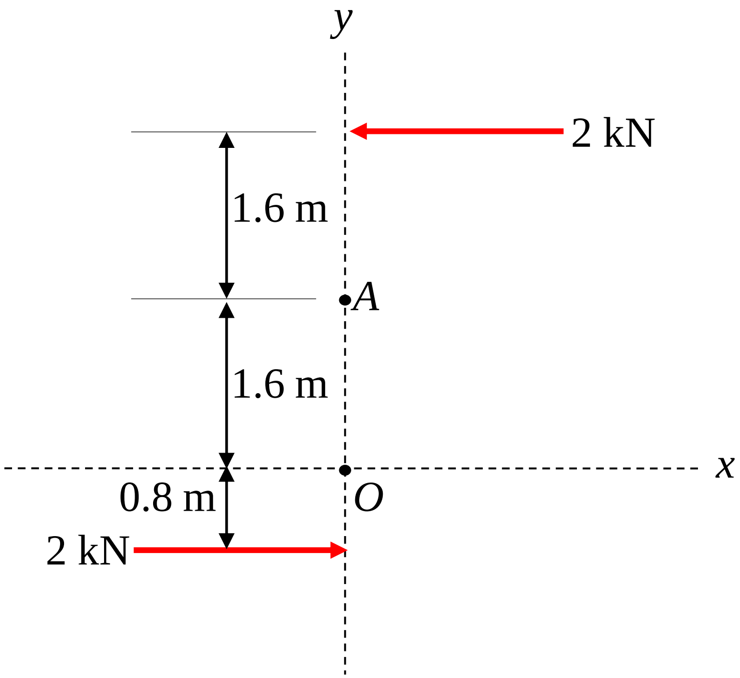


Problem-1

- Calculate the combined moment of the two 2-kN forces, shown in the Figure, about point *O* and A using :

(i) Scalar approach

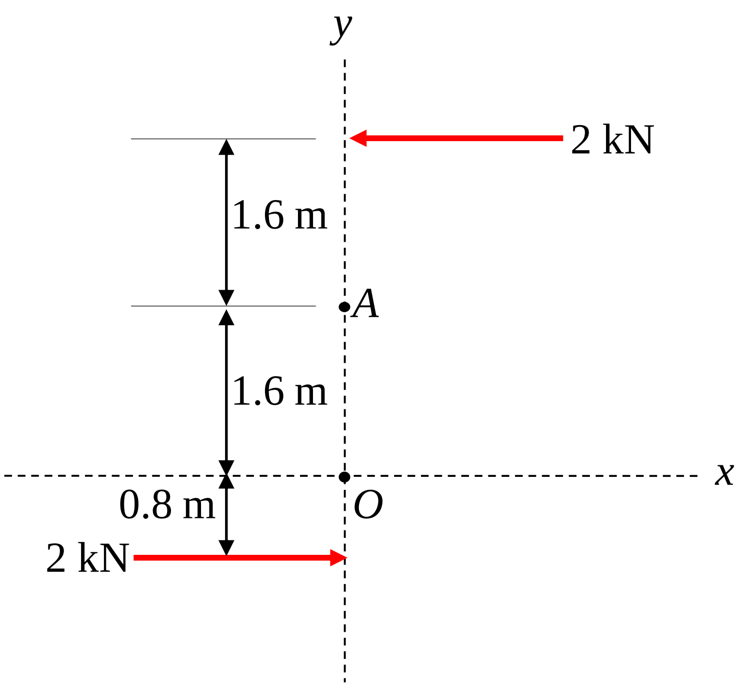
(ii) Vector approach



Problem-1-Continous

i-

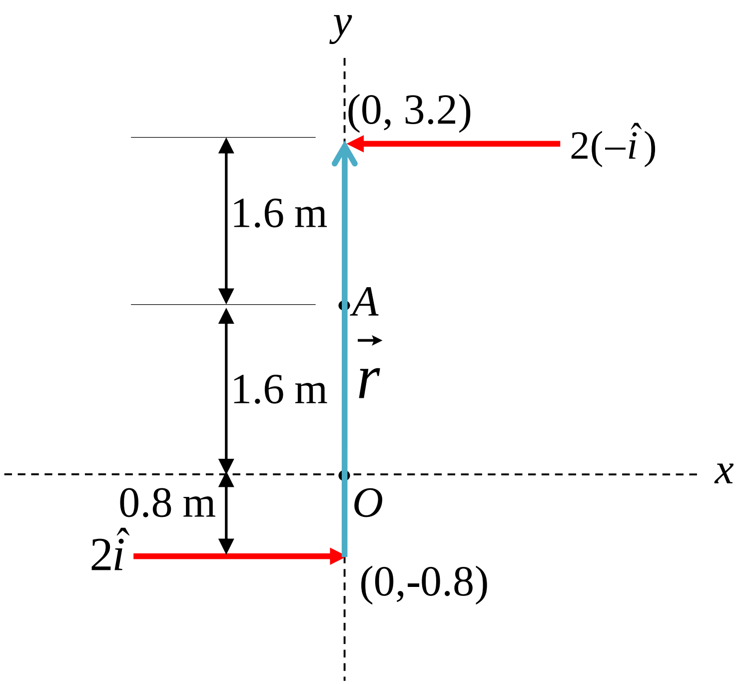
As 2 kN forces are forming a couple, moments will be the same about *A* and *O* as couple moment is independent of moment centres.



$$M_O = M_A = Fd = 2\times(1.6+1.6+0.8) = 8\,\text{kN.m}$$

ii-

As 2 kN forces are forming a couple, moments will be the same about *A* and *O* as couple moment is independent of moment centres.



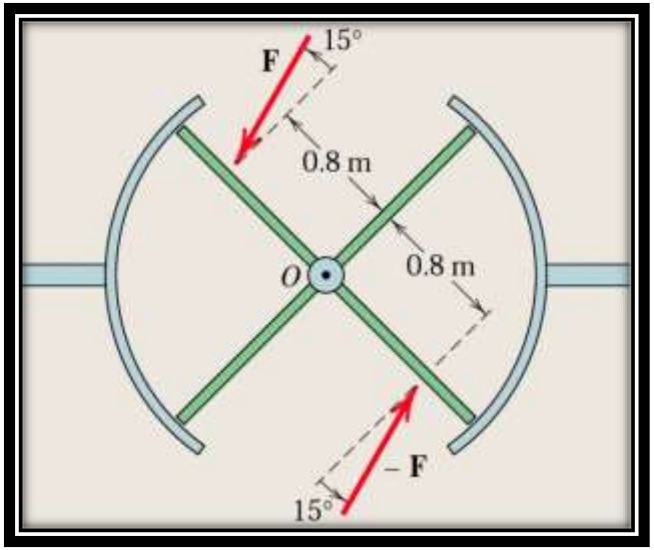
$$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

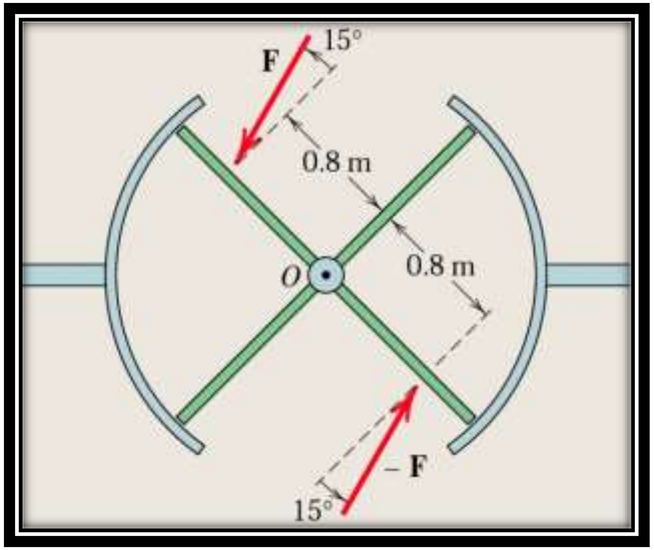
$$\Rightarrow \vec{r} = (0-0)\hat{i} + (3.2 - (-0.8))\hat{j} = 4\hat{j} \text{ m}$$

$$\text{Therefore } \overrightarrow{M}_O = \overrightarrow{M}_A = \vec{r} \times \vec{F} = 4\hat{j} \times 2(-\hat{i}) = 8\hat{k} \text{ kN.m}$$

Problem-2

The top view of a revolving entrance door is shown in the Figure. Two persons simultaneously approach the door and exert forces of equal magnitude as shown. If the resulting moment about point O is 25 Nm, determine the force magnitude *F*.





The two components $F\cos 15^{\circ}$ and -$F\cos 15^{\circ}$ will form a couple. The moment of this couple would be

$M_O = F\cos 15^{\mathrm{o}} \times 1.6 = 25\ \mathrm{Nm}$

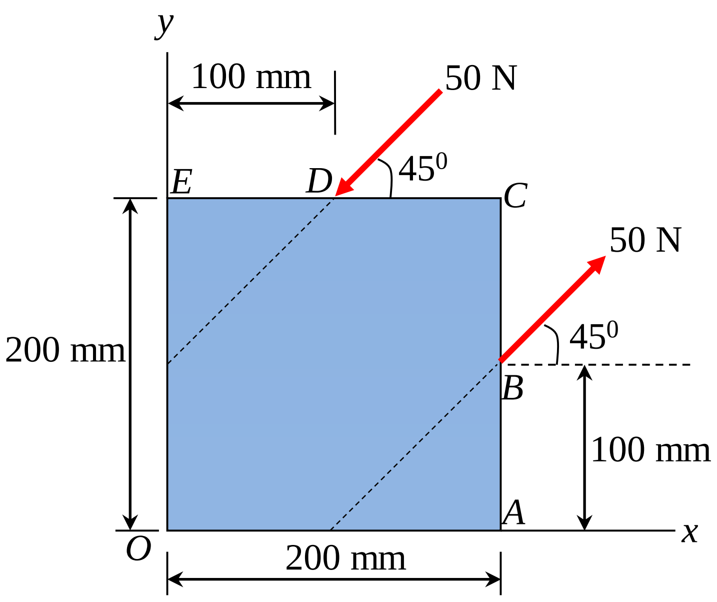
$\Rightarrow F = 16.18\ \mathrm{N}$

Note: The components $F\sin 15^{\circ}$ and -$F\sin 15^{\circ}$ will not cause any moment as their lines of action are paasing through O.

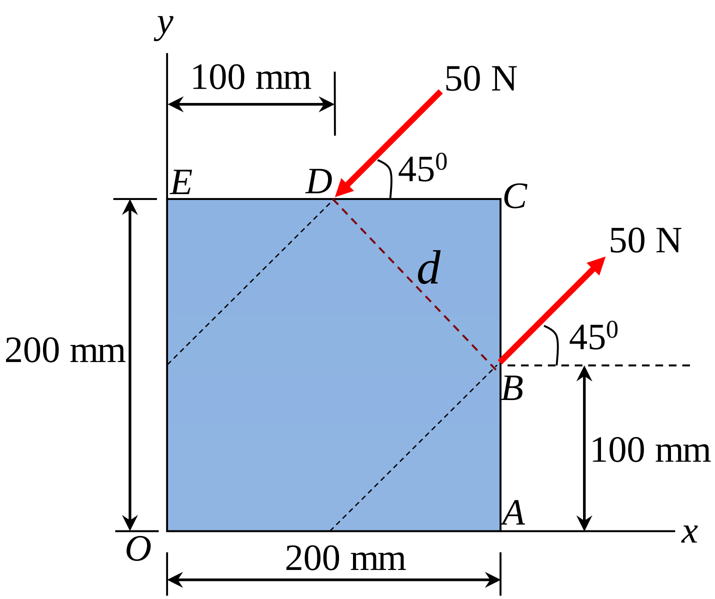
Problem-3

A square plate of 200 mm × 200 mm is subjected to two forces, each of magnitude 50 N, as shown in the figure:

1. Calculate the moment of the forces about points *O*, *A*, *C*, and *D*.
2. Find the moment of the forces about *y*-axis.



1-

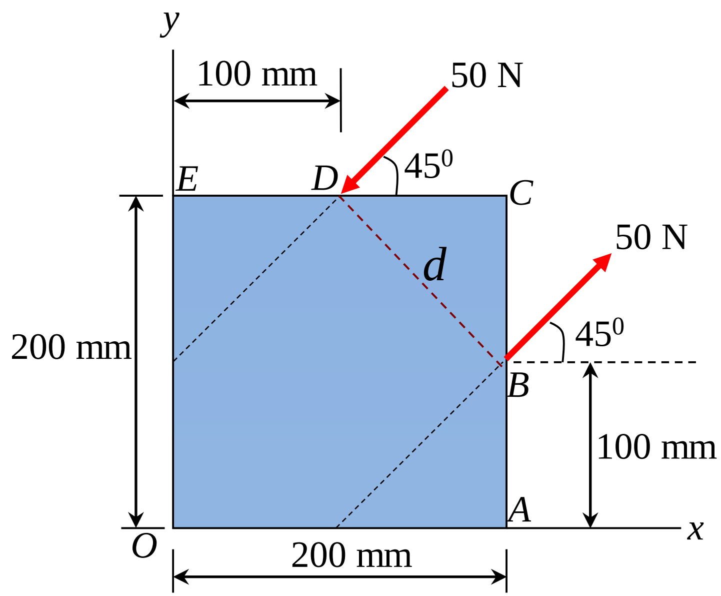


The perpendicular distance d between two 50 N forces $= \sqrt{100^2 + 100^2} = 141.4$ mm

Since two 50 N forces form a couple. The moment of this couple is $M = Fd = 50 \times 141.4 = 7070$ N.mm

As couple is independent of moment centers, $M_O = M_A = M_C = M_D = M = 7070$ N.mm *Ans.*

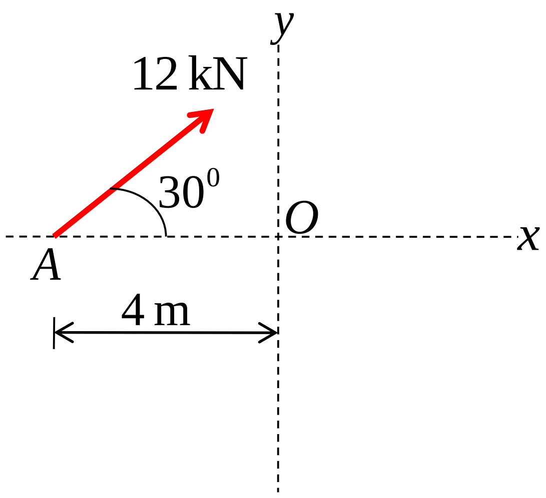
2-

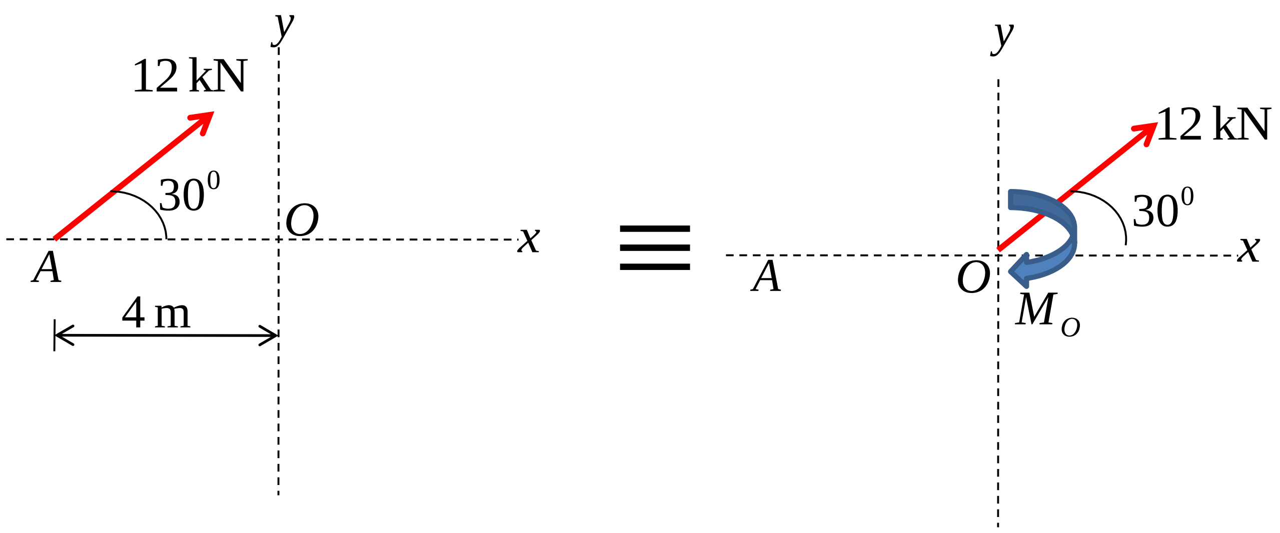


Since forces are acting in xy plane, they will either intersect the axes or they will be parallel to them. Therefore, moment about x- or y-axis of all the coplanar forces = 0

Problem-4

Replace the 12-kN force acting at point *A* by a force-couple system acting at point *O* .





$$+ \quad M_O = -12 \times d = -12 \times 4 \sin 30^0 = -24 \text{ kNm}$$