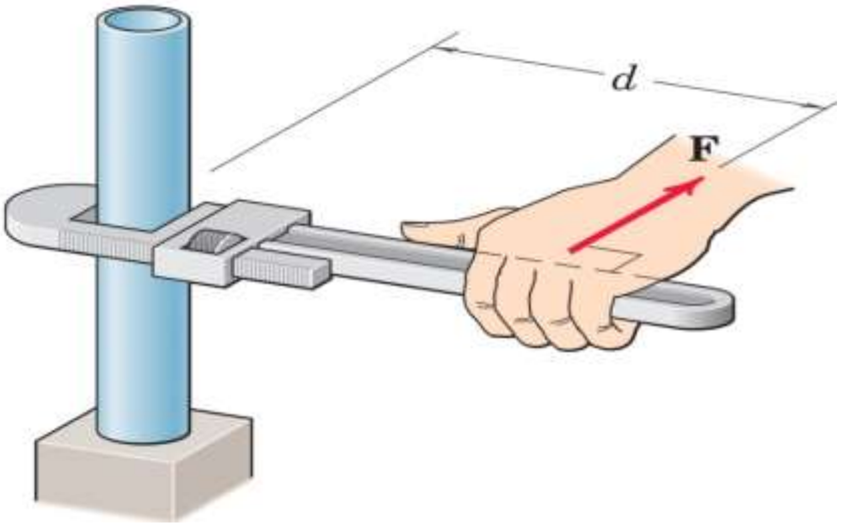
Engineering Mechanics(Statics)
Lecture 4

Moment

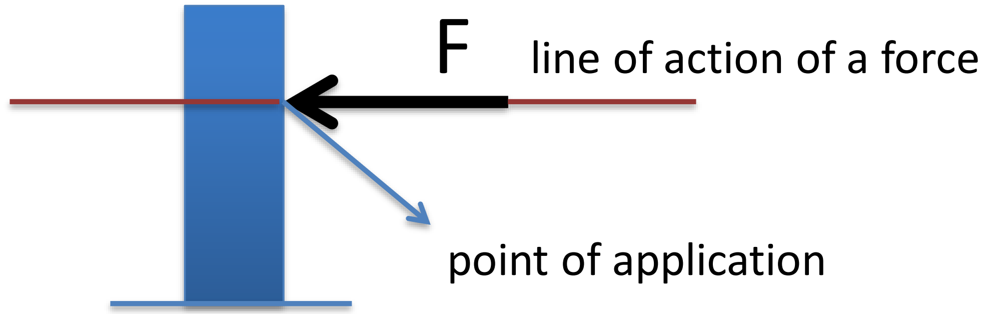
Moment

- In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis.
- The axis may be any line which neither intersects nor is parallel to the line of action of the force.
- M=Fd



Moment about a Point

- Point of application of a force and line of action of a force are as in Fig:



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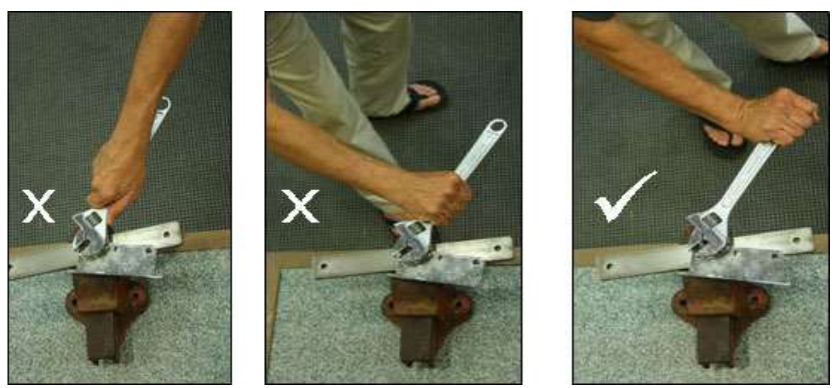
- The magnitude of the moment or tendency of the force to rotate the body about the axis perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm d*, which is the perpendicular distance from the axis to the line of action of the force.

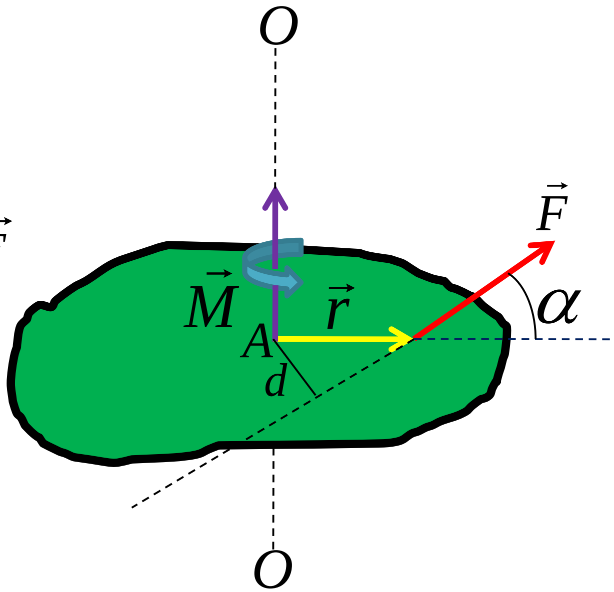


Moment: Vector Definition

- In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations.

$$\vec{M} = \vec{r} \times \vec{F}$$

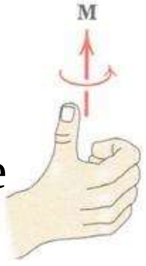
$\vec{r}$ = Position vector that runs from the moment reference point A to *any point* the line of action of $\vec{F}$



$$\text{Magnitude of } \vec{M} = \left|\vec{M}\right| = M = rF \sin\alpha$$

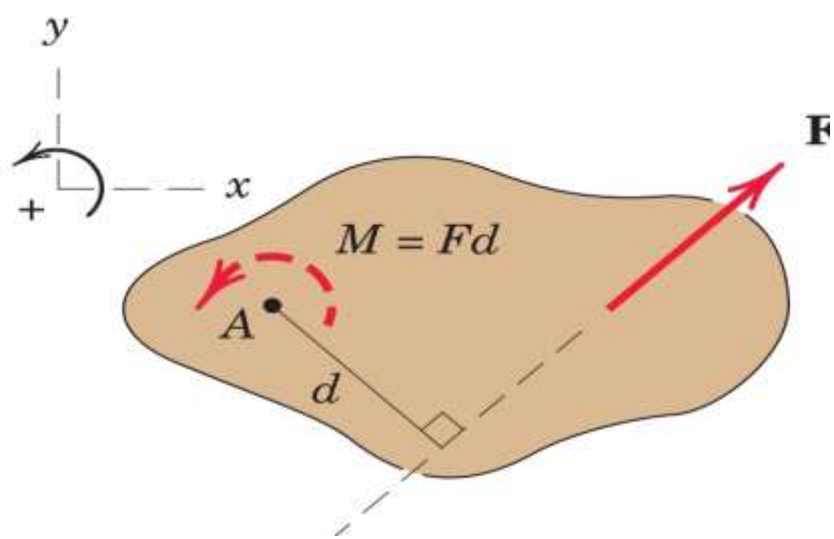
$$\Rightarrow M = Fr\sin\alpha = Fd$$

Direction: The right-hand rule is used to identify the direction. Thus the moment of $\vec{F}$ about O-O may be identified as a vector pointing in the direction of thumb, with the fingers curled in the direction of the tendency to rotate.



Clockwise and Counterclockwise moments:

- If tendency of the force is to rotate its *moment arm* in a counter clockwise manner it is called counterclockwise moment (CCW). We will assume CCW moment as a positive moment (+).
- If tendency of the force is to rotate its *moment arm* in a clockwise manner it is called clockwise moment (CW). We will assume CW moment as a negative moment (-).



Zero moments

- *If line of action of the force intersects the axis of rotation*, there is no moment of the force about this axis of rotation
- *If line of action of the force is parallel to the axis of rotation*, there is no moment of the force about this axis of rotation

Varignon's Theorem

- The moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

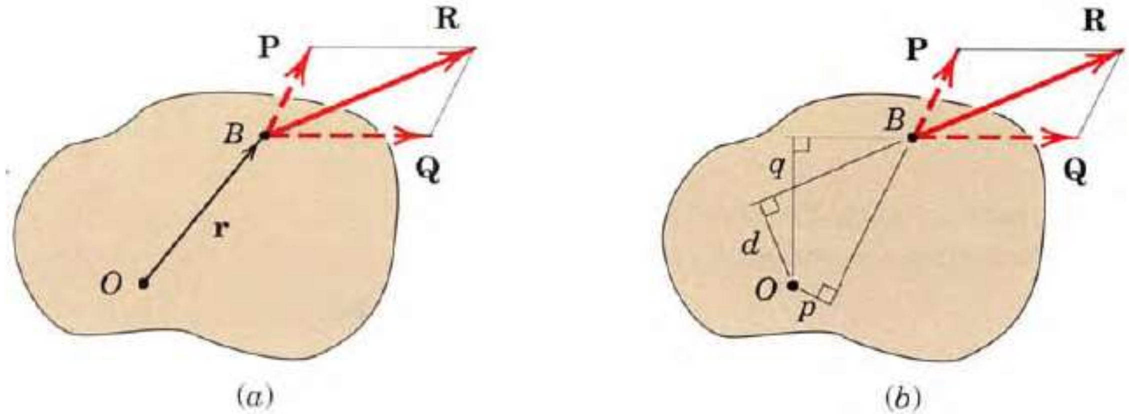


Figure 2/9

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

Figure 2/9*b* illustrates the usefulness of Varignon's theorem. The moment of **R** about point O is Rd. However, if d is more difficult to determine than p and q, we can resolve **R** into the components **P** and **Q**, and compute the moment as

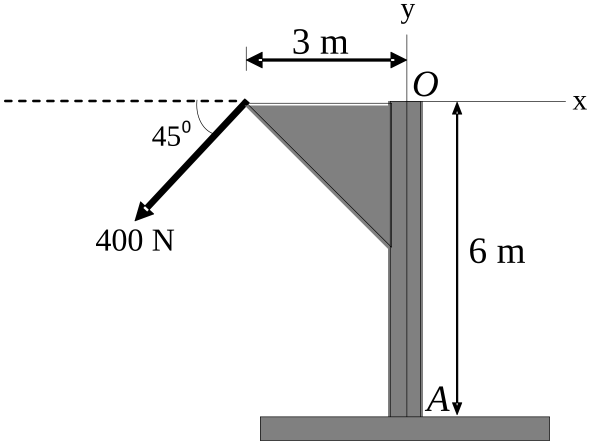
$$M_O = Rd = -pP + qQ$$

where we take the clockwise moment sense to be positive.

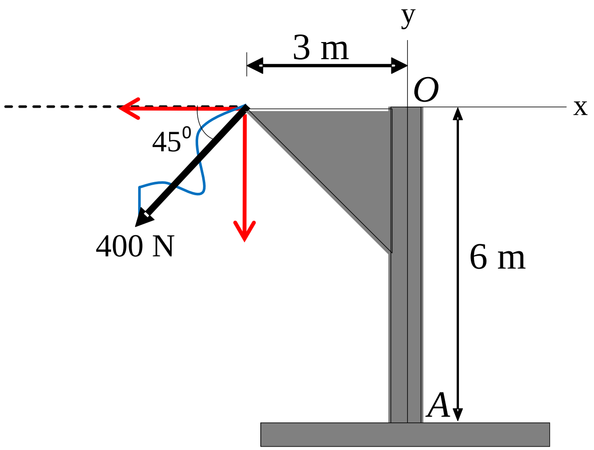
Problems

For the figure shown below, find moment of 400 N force about point *A* using:

1. Scalar approach
2. vector approach.



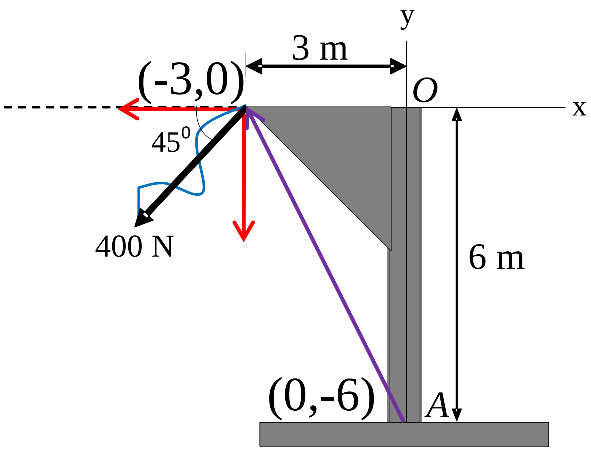
Scalar Approach



$$M_A = 400\cos 45^0 \times 6 + 400\sin 45^0 \times 3$$

$$\Rightarrow M_A = 2545.2\ \text{N.m (Counterclock wise)} \quad Ans.$$

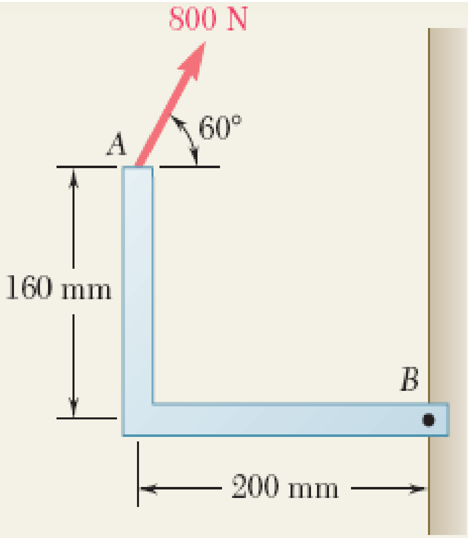
Vector Approach



$$\vec{F} = 400\cos 45^0(-\hat{i}) + 400\sin 45^0(-\hat{j}) = -282.8\hat{i} - 282.8\hat{j}$$

$$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} = (-3-0)\hat{i} + (0-(-6))\hat{j} = -3\hat{i} + 6\hat{j}$$

$$\vec{M}_A = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 6 & 0 \\ -282.8 & -282.8 & 0 \end{vmatrix} = 2545.2\hat{k} \quad \text{N.m} \qquad \textit{Ans.}$$



SAMPLE PROBLEM 3.2

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.

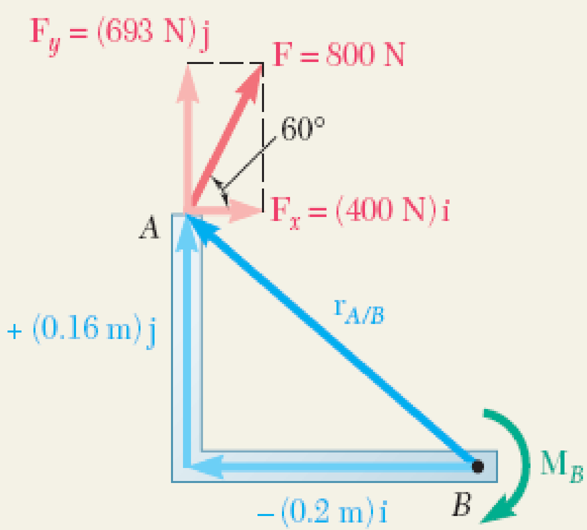
SOLUTION

The moment $\mathbf{M}_B$ of the force $\mathbf{F}$ about B is obtained by forming the vector product

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}$$

where $\mathbf{r}_{A/B}$ is the vector drawn from B to A. Resolving $\mathbf{r}_{A/B}$ and $\mathbf{F}$ into rectangular components, we have

$$\begin{aligned} \mathbf{r}_{A/B} &= -(0.2 \text{ m})\mathbf{i} + (0.16 \text{ m})\mathbf{j} \\ \mathbf{F} &= (800 \text{ N}) \cos 60°\mathbf{i} + (800 \text{ N}) \sin 60°\mathbf{j} \\ &= (400 \text{ N})\mathbf{i} + (693 \text{ N})\mathbf{j} \end{aligned}$$



Recalling the relations (3.7) for the cross products of unit vectors (Sec. 3.5), we obtain

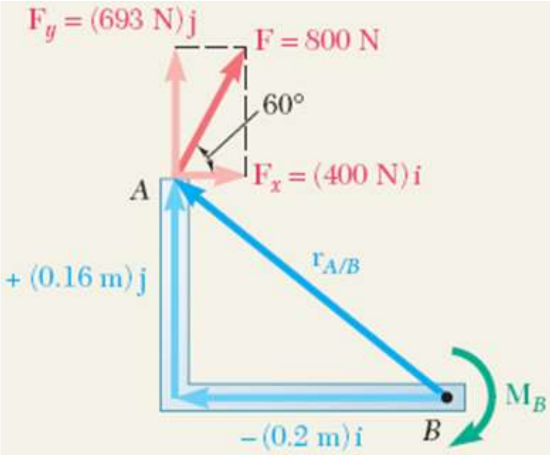
$$\begin{aligned} \mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} &= [-(0.2 \text{ m})\mathbf{i} + (0.16 \text{ m})\mathbf{j}] \times [(400 \text{ N})\mathbf{i} + (693 \text{ N})\mathbf{j}] \\ &= -(138.6 \text{ N} \cdot \text{m})\mathbf{k} - (64.0 \text{ N} \cdot \text{m})\mathbf{k} \\ &= -(202.6 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

$$\mathbf{M}_B = 203 \text{ N} \cdot \text{m} \circlearrowright$$ ◀

The moment $\mathbf{M}_B$ is a vector perpendicular to the plane of the figure and pointing *into* the paper.

Alternative Solution (Using Varignon's Theorem):

Assuming that the counterclockwise moment is positive;



$M_B = - F_x.(0.16m) - F_y.(0.2m)$

where $F_x = 400$ N and $F_y = 693$ N

$M_B = -400N(0.16m)-693N(0.2m) = -202.6$ N.m

We can say $M_B = -203$ N.m

Since we assumed that the counterclockwise moment is positive and we obtained a negative moment value, this implies that the direction of M_B is actually clockwise.

So; the result is exactly the same as before, i.e.