Force Systems

- Force: Action of one body on another.
  - Specification include:
    - Magnitude
    - Direction
    - Point of application (or line of action)
- Effects of a force

External: ⟹ Reactions

Internal: ⟹ Stresses and strains

In dealing with the mechanics of rigid bodies concern is only to the net <u>external effects</u> of forces.

CHAPTER 2 – FORCE SYSTEMS

(RECTANGULAR COMPONENTS)

2/3 Rectangular Components

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector **F** of Fig. 2/5 may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \qquad \mathbf{(2/1)}$$

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of **F** in the *x*- and *y*-directions.

$\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \qquad \mathbf{(2/2)}$$

where the scalars F_x and F_y are the x and y *scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig. 2/5, the x and y scalar components are both positive and are related to the magnitude and direction of **F** by

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$
$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x} \qquad \mathbf{(2/3)}$$

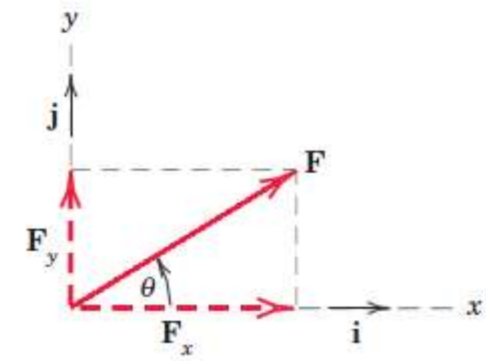
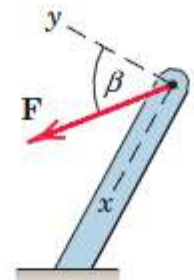
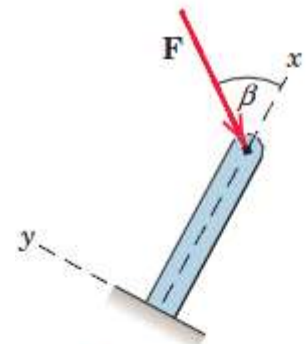


Figure 2/5



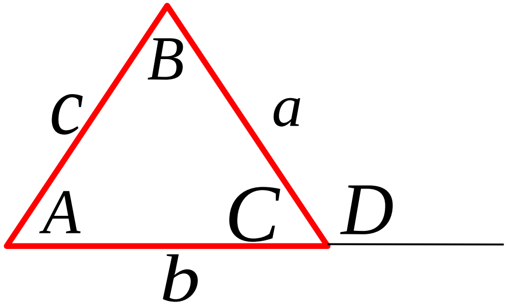
$F_x = F\sin\beta$
$F_y = F\cos\beta$



$F_x = -F\cos\beta$
$F_y = -F\sin\beta$

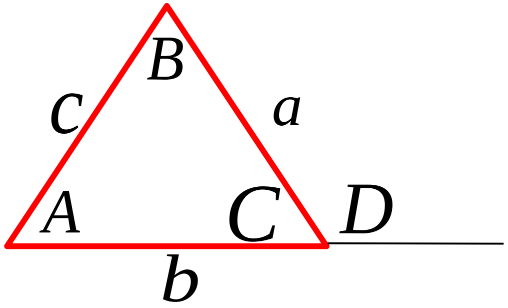
Law of sines and cosines

Law of sines



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines

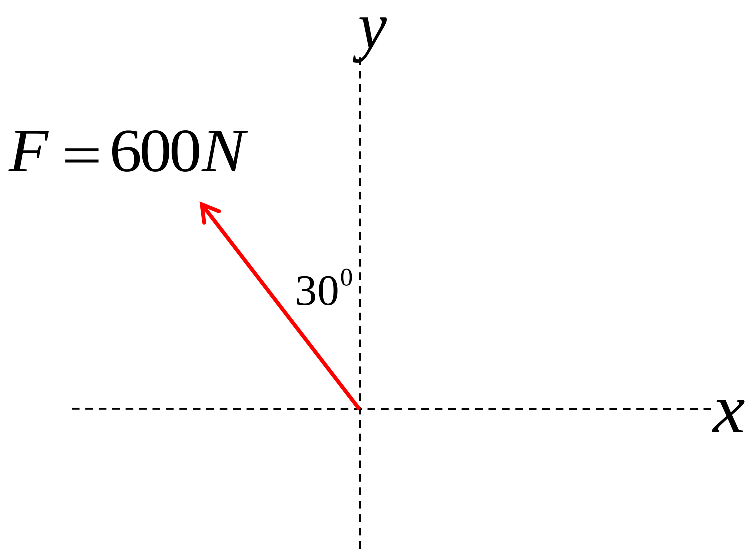


$$c^2 = a^2 + b^2 - 2ab\cos C$$

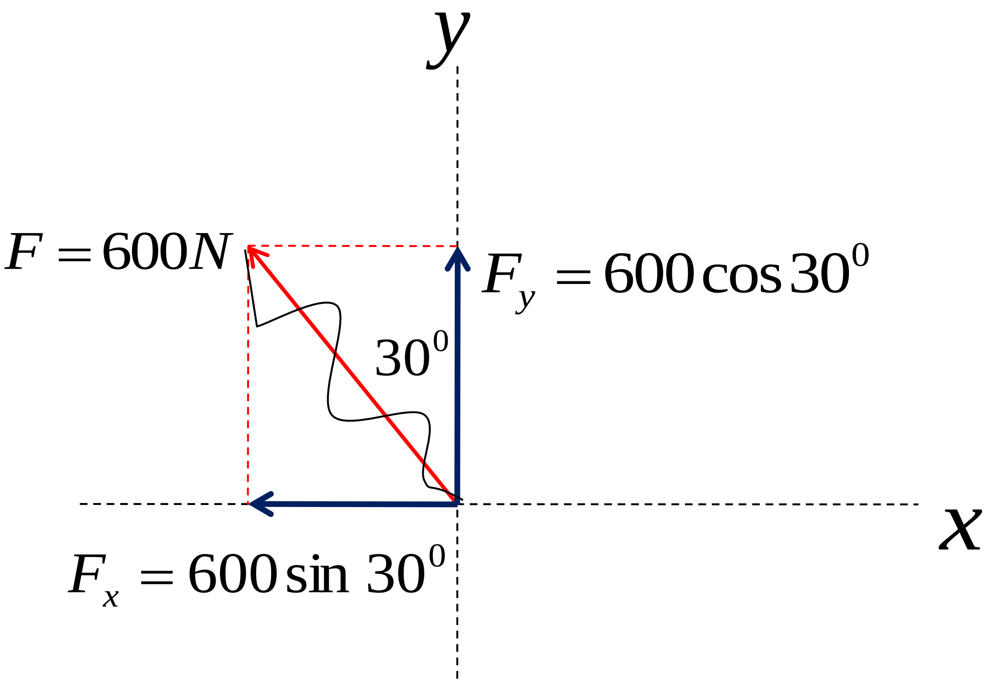
$$c^2 = a^2 + b^2 + 2ab\cos D$$

Problem-1

The force F has a magnitude of 600 N. Express F as a vector in terms of the unit vectors $\vec{i}$ and $\vec{j}$. Identify the x and y scalar components of $\vec{F}$.



Solution



$$\vec{F}_x = 600\sin 30^0(-\vec{i})$$

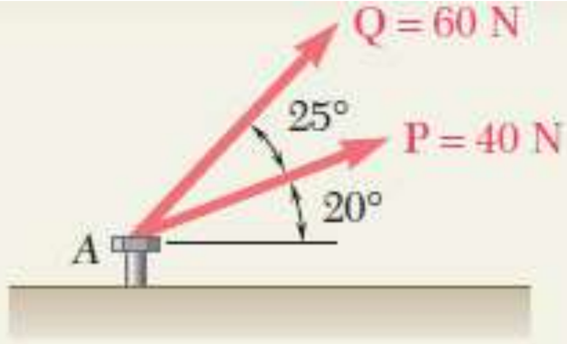
$$\vec{F}_y = 600\cos 30^0\,\vec{j}$$

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F} = -300\vec{i} + 520\vec{j} \quad N$$

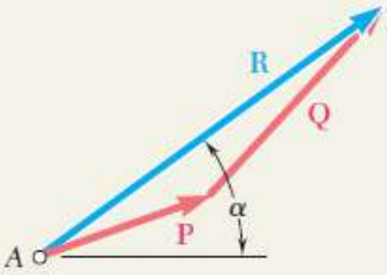
CHAPTER 2 – FORCE SYSTEMS

EXAMPLE - 1



The two forces **P** and **Q** act on a bolt *A*.

Determine their resultant.



Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

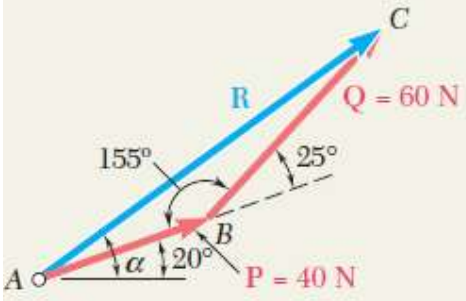
$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$R^2 = (40 \text{ N})^2 + (60 \text{ N})^2 - 2(40 \text{ N})(60 \text{ N}) \cos 155°$$
$$R = 97.73 \text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60 \text{ N}} = \frac{\sin 155°}{97.73 \text{ N}} \tag{1}$$

Solving Eq. (1) for sin *A*, we have

$$\sin A = \frac{(60 \text{ N}) \sin 155°}{97.73 \text{ N}}$$



Using a calculator, we first compute the quotient, then its arc sine, and obtain

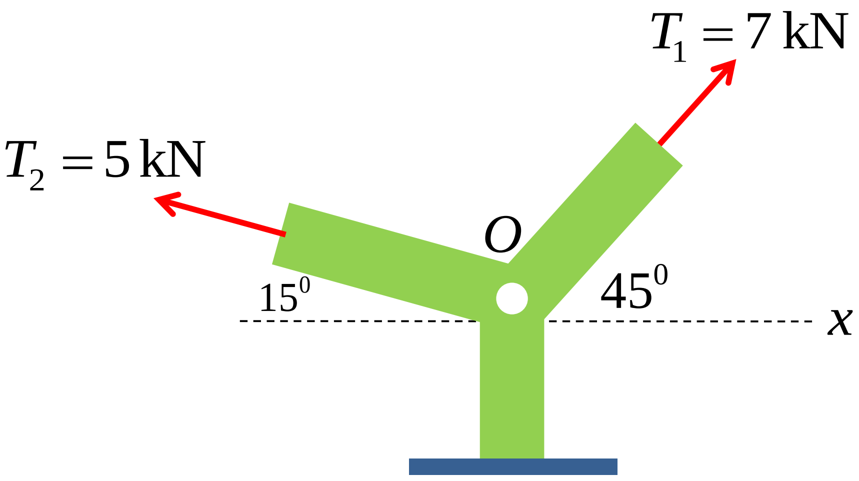
$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

We use 3 significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7 \text{ N} \measuredangle 35.0°$$

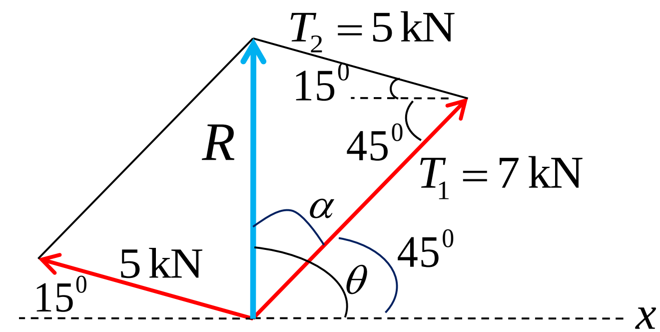
Problem-2

Determine the magnitude of the resultant *R* of the two forces (shown below), and the angle *ϑ* which *R* makes with the positive *x*-axis.



Solution

This problem can be viewed how two non-rectangular force components can be replaced by a single resultant force R.

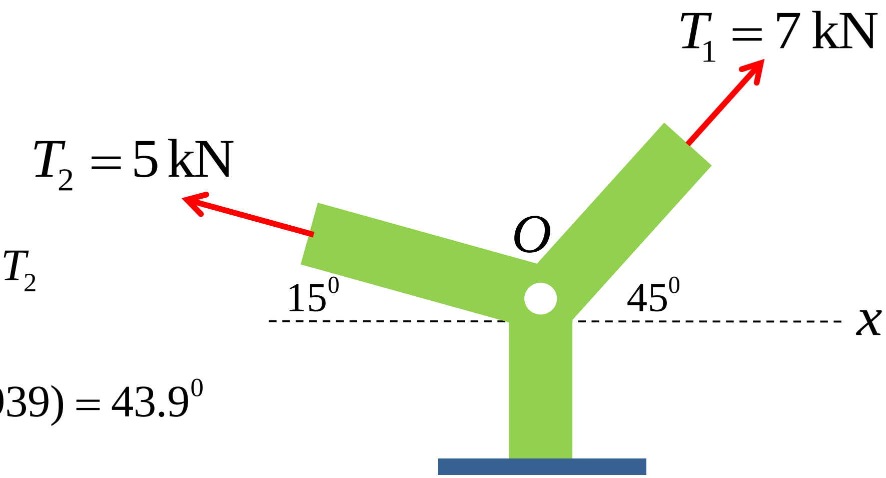


Use cosine law : $c^2 = a^2 + b^2 - 2ab\cos C$

$$\therefore R = \sqrt{T_1^2 + T_2^2 - 2T_1 \times T_2 \times \cos(45^0 + 15^0)}$$

$$= \sqrt{7^2 + 5^2 - 2 \times 5 \times 7\cos(45^0 + 15^0)}$$

$$= 6.24\,\text{kN}$$



$$\frac{\sin\alpha}{T_2} = \frac{\sin(45^0 + 15^0)}{R} \Rightarrow \sin\alpha = \frac{\sin(45^0 + 15^0)}{R} \times T_2$$

$$\Rightarrow \sin\alpha = \frac{\sin(60^0)}{6.24} \times 5 = 0.6939 \Rightarrow \alpha = \sin^{-1}(0.6939) = 43.9^0$$

$$\therefore \theta = \alpha + 45^0 = 43.9^0 + 45^0 = 88.9^0$$

Alternative Solution

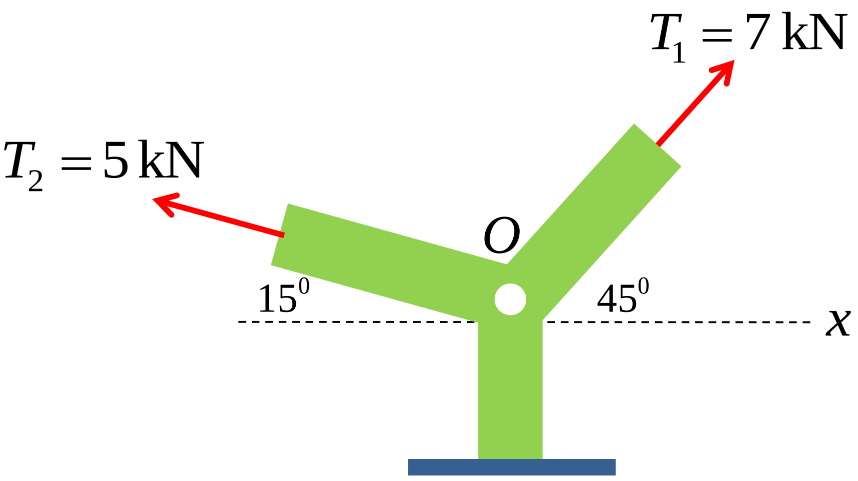
Note: This method will be discussed in *Force Resultants* topic.

$$R_x = \sum F_x = T_1 \cos 45^0 - T_2 \cos 15^0 = 7\cos 45^0 - 5\cos 15^0 = 0.12\,\text{kN}$$

$$R_y = \sum F_y = T_1 \sin 45^0 + T_2 \sin 15^0 = 7\sin 45^0 + 5\sin 15^0 = 6.24\,\text{kN}$$

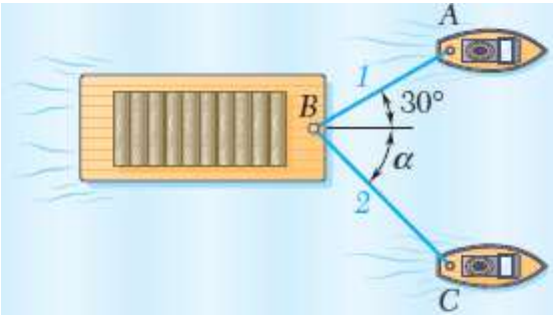
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{0.12^2 + 6.24^2} = 6.24\,\text{kN}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{6.24}{0.12}\right) = 88.9^0$$

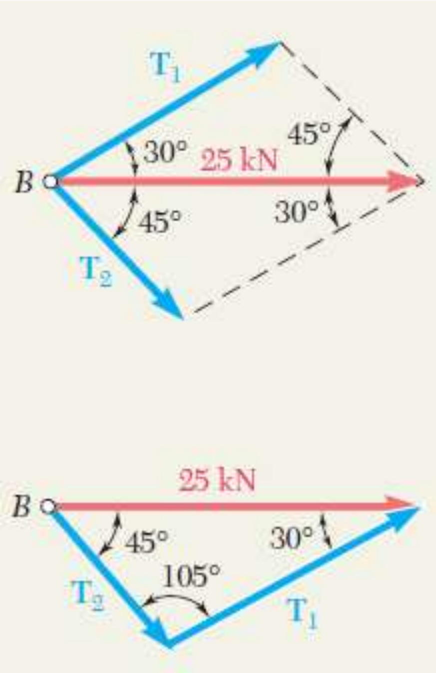


CHAPTER 2 – FORCE SYSTEMS

EXAMPLE - 2



A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 25 kN force directed along the axis of the barge, determine (*a*) the tension in each of the ropes knowing that $\alpha = 45°$, (*b*) the value of α for which the tension in rope 2 is minimum.



(*a*) the tension in each of the ropes knowing that $\alpha = 45°$

Trigonometric Solution. The triangle rule can be used. We note that the triangle shown represents half of the parallelogram shown above. Using the law of sines, we write

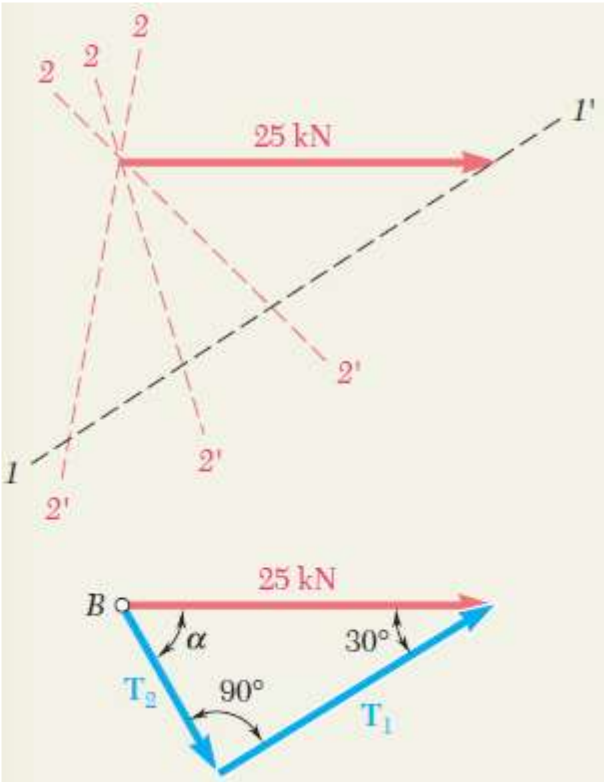
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{25 \text{ kN}}{\sin 105°}$$

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 45° and sin 30°, we obtain

$$T_1 = 18.3 \text{ kN} \qquad T_2 = 12.94 \text{ kN}$$

CHAPTER 2 – FORCE SYSTEMS

EXAMPLE – 2 (CONTINUED)



b. Value of α for Minimum T_2. To determine the value of α for which the tension in rope *2* is minimum, the triangle rule is again used. In the sketch shown, line *1-1'* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2'*. We note that the minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular. The minimum value of T_2 is

$$T_2 = (25 \text{ kN}) \sin 30° = 12.5 \text{ kN}$$

Corresponding values of T_1 and α are

$$T_1 = (25 \text{ kN}) \cos 30° = 21.7 \text{ kN}$$

$$\alpha = 90° - 30° \qquad \alpha = 60°$$