Contents

- Objective(s) of the present lecture (#2)
- Quantities: Scalars and Vectors
- Vector operations

Objectives of the Present lecture

- *To define scalar and vector quantities*
- *To provide an overview of most common vector operations*

SCALAR QUANTITIES

A scalar is a quantity that has only magnitude, either positive or negative.

For example; mass, length, area, volume and speed are the scalar quantities frequently used in Statics.

Scalars are indicated by letters in *Itallic* type, such as the scalar ‘*A*’.

VECTOR QUANTITIES

A vector is a quantity that has both a magnitude and a direction.

For example; weight, force, moment, position, velocity and acceleration are the vector quantities frequently used in Statics.

Vectors are indicated by bold letters, such as '**A**' or $\overrightarrow{\mathbf{A}}$

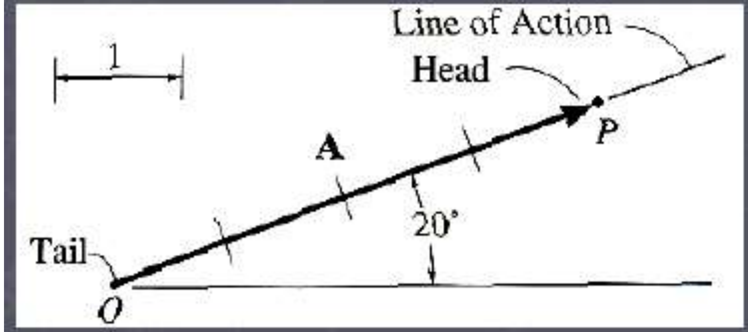
The magnitude of a vector is always a positive quantity and is symbolized in *Itallic* type, written as *A* or |A|

Scalars and Vectors

Vector Quantities----contd.

- A vector is **represented graphically by an arrow**, which is used to define its
  - magnitude (by the **length** of the arrow)
  - direction (by the **angle** between a reference axis and the arrow's line of action)
  - sense (by the **arrowhead**)

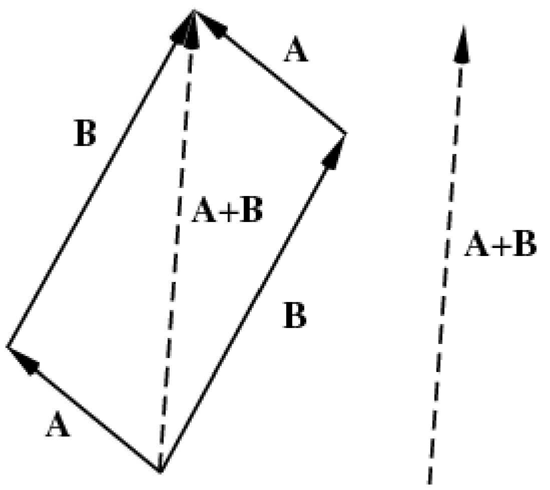
For **example**, the vector shown below has



- A **magnitude** = 4 units
- A **direction** = 20°, measured counterclockwise from the horizontal axis
- A **sense** which is upward and to the right
- Point **O is called *tail*** of the vector
- Point **P is called *tip or head*** of the vector

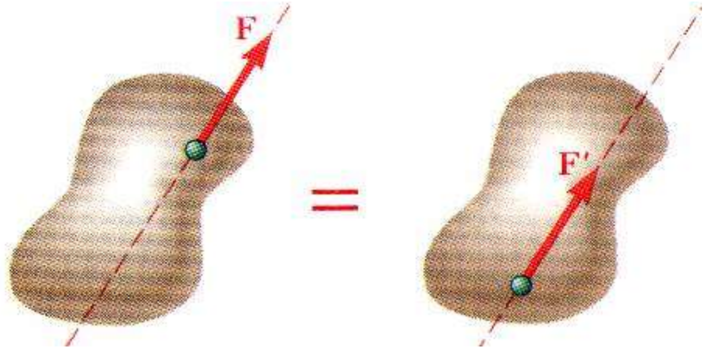
Fundamental Principles – Paralellogram Law & Transmissibility

- ***Parallelogram Law***



Vectors must obey the parallelogram law of combination. This law states that two vectors A and B, treated as free vectors, may be replaced by their equivalent vector (A+B), which is the diagonal of the parelellogram formed by A and B as its two sides, as shown in the figure.

- ***Principle of Transmissibility***
  Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action.
  NOTE: **F** and **F**' are equivalent forces.



VECTOR OPERATIONS

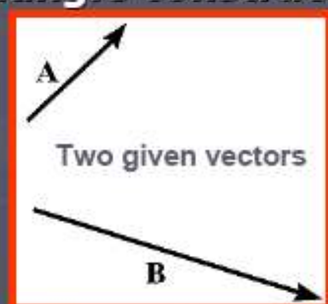
Vector Addition

Two vectors **A** and **B** may be added to form a resultant vector

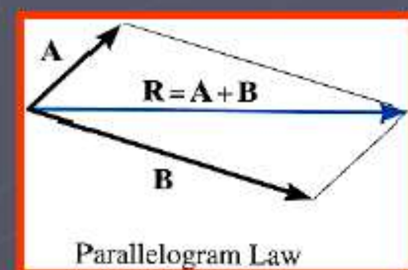
$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

using the following methods:

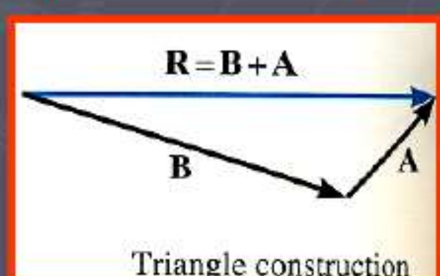
- Parallelogram law
- Triangle construction



Two given vectors

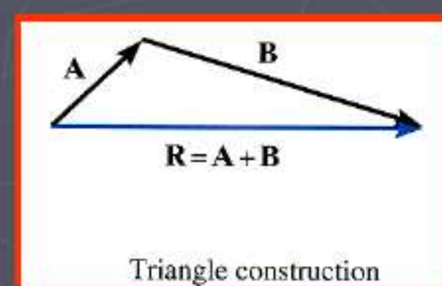


Parallelogram Law



Triangle construction

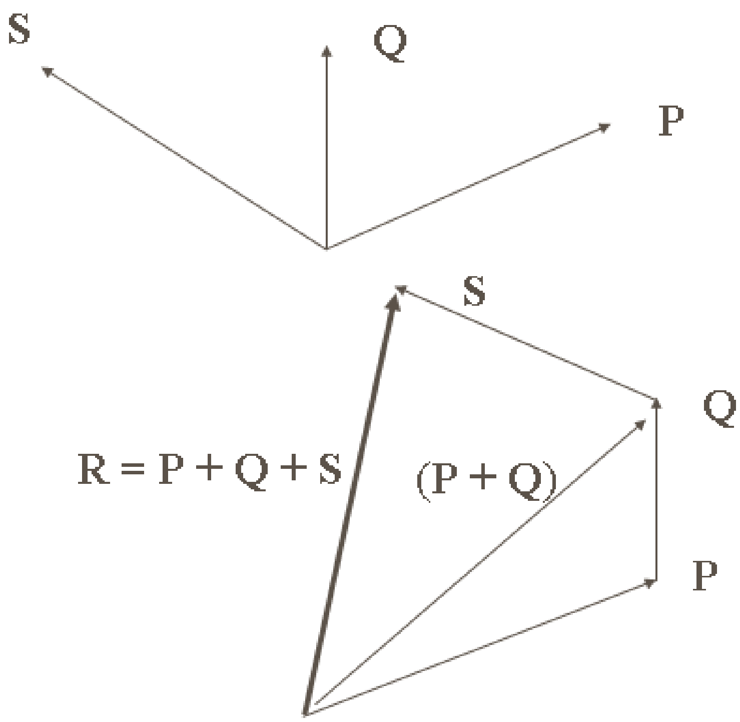
or

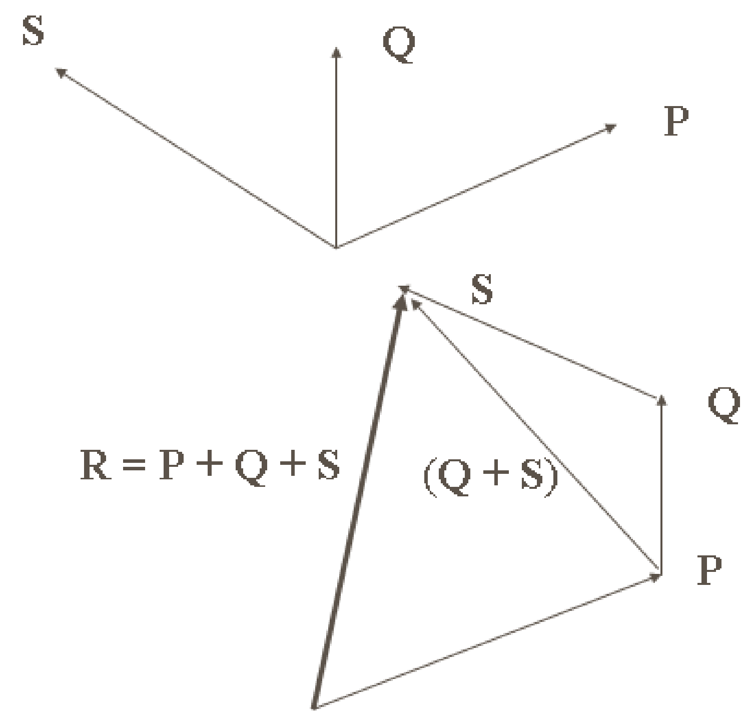


Triangle construction

Polygon Rule:

can be used for the addition of more than two vectors. Two vectors are actually summed and added to the third and so on...





P + Q + S = (P + Q) + S = P + (Q + S)

This is the Associative Law of Vector Addition

VECTOR OPERATIONS

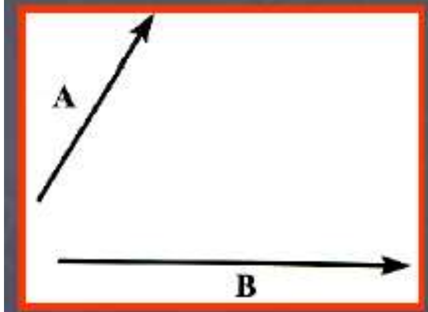
Vector Subtraction

- Vectors **A** and **B** may be subtracted to form a resultant vector

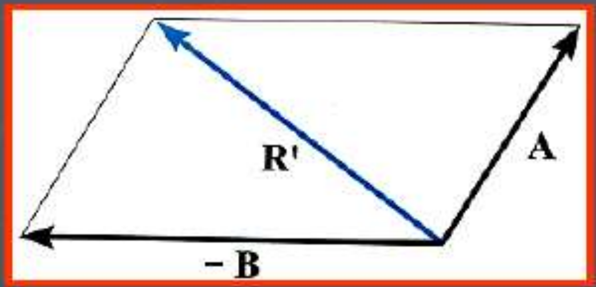
$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

using the following methods:

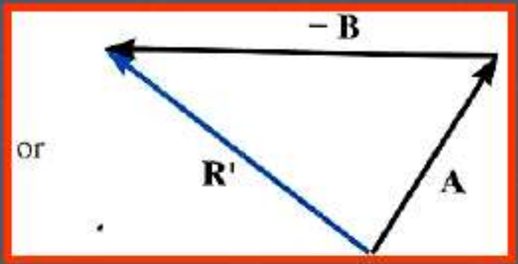
- Parallelogram law
- Triangle construction



Two given vectors



Parallelogram law

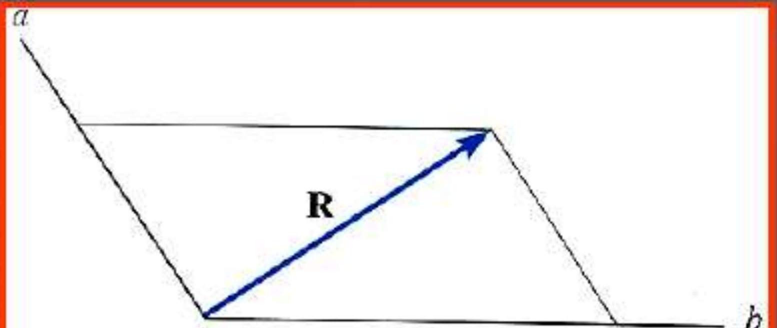


Triangular construction

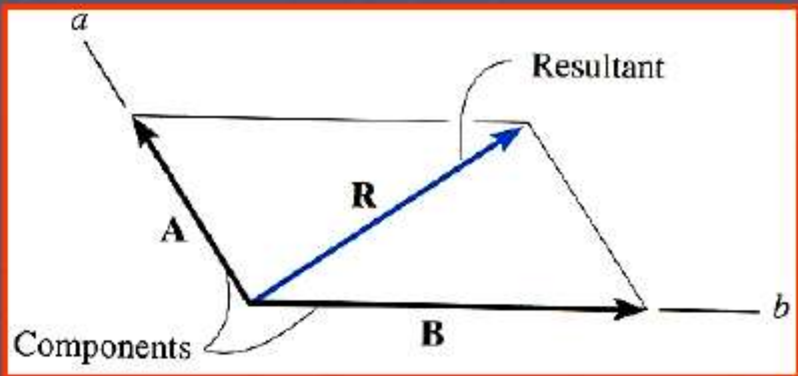
VECTOR OPERATIONS

Resolution of Vector

- A vector may be resolved into two components having known lines of action by using the parallelogram law.
- For example, a vector **R** may be resolved into two vectors **A** and **B** along the lines a and b, using the parallelogram law, as shown below:

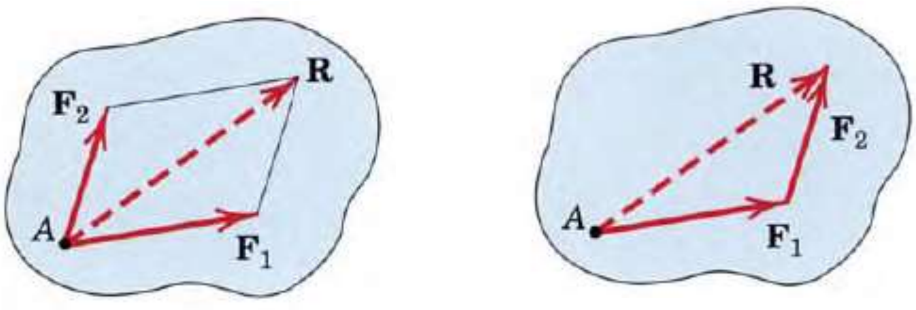


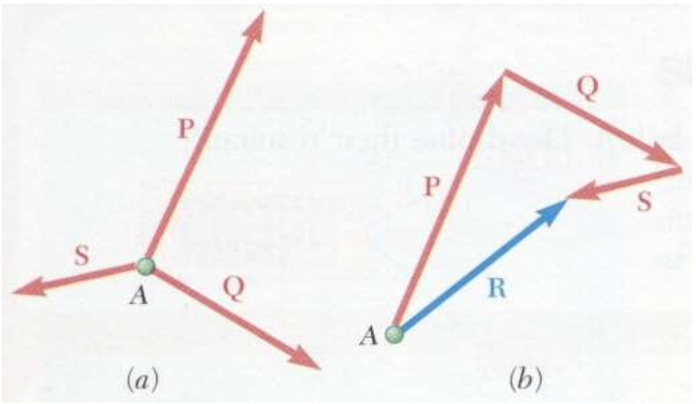
Extend parallel lines from the head of **R** to form components



CHAPTER 2 – FORCE SYSTEMS

(RESULTANT OF VECTORS)





- *Concurrent forces*: set of forces which all pass through the same point.

  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.

Conventions for Equations and Diagrams

A vector quantity **V** is represented by a line segment, Fig. 1/1, having the direction of the vector and having an arrowhead to indicate the sense. The length of the directed line segment represents to some convenient scale the magnitude $|\mathbf{V}|$ of the vector and is printed with lightface italic type V.

In scalar equations, and frequently on diagrams where only the magnitude of a vector is labeled, the symbol will appear in lightface italic type. Boldface type is used for vector quantities whenever the directional aspect of the vector is a part of its mathematical representation. When writing vector equations, *always* be certain to preserve the mathematical distinction between vectors and scalars. In handwritten work, use a distinguishing mark for each vector quantity, such as an underline, $\underline{V}$, or an arrow over the symbol, $\vec{V}$, to take the place of boldface type in print.

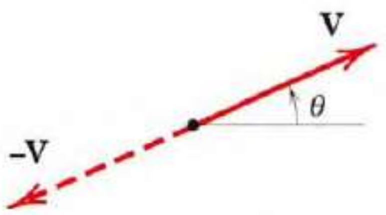


Figure 1/1

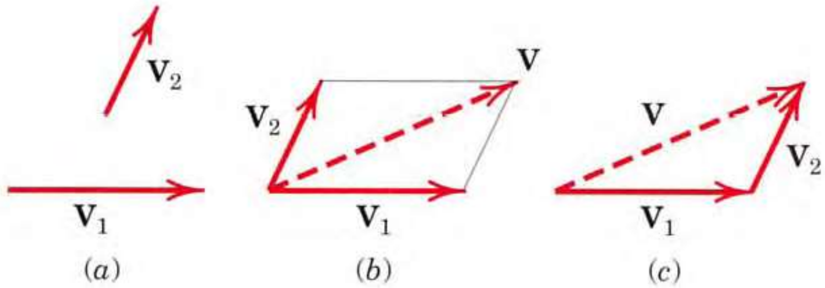


Figure 1/2

Working with Vectors

The direction of the vector **V** may be measured by an angle θ from some known reference direction as shown in Fig. 1/1. The negative of **V** is a vector $-\mathbf{V}$ having the same magnitude as **V** but directed in the sense opposite to **V**, as shown in Fig. 1/1.

Vectors must obey the parallelogram law of combination. This law states that two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, treated as free vectors, Fig. 1/2*a*, may be replaced by their equivalent vector **V**, which is the diagonal of the parallelogram formed by $\mathbf{V}_1$ and $\mathbf{V}_2$ as its two sides, as shown in Fig. 1/2*b*. This combination is called the *vector sum*, and is represented by the vector equation

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

where the plus sign, when used with the vector quantities (in boldface type), means *vector* and not *scalar* addition. The scalar sum of the magnitudes of the two vectors is written in the usual way as $V_1 + V_2$. The geometry of the parallelogram shows that $V \neq V_1 + V_2$.

The two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, again treated as free vectors, may also be added head-to-tail by the triangle law, as shown in Fig. 1/2*c*, to obtain the identical vector sum $\mathbf{V}$. We see from the diagram that the order of addition of the vectors does not affect their sum, so that $\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1$.

The difference $\mathbf{V}_1 - \mathbf{V}_2$ between the two vectors is easily obtained by adding $-\mathbf{V}_2$ to $\mathbf{V}_1$ as shown in Fig. 1/3, where either the triangle or parallelogram procedure may be used. The difference $\mathbf{V}'$ between the two vectors is expressed by the vector equation

$$\mathbf{V}' = \mathbf{V}_1 - \mathbf{V}_2$$

where the minus sign denotes *vector subtraction*.

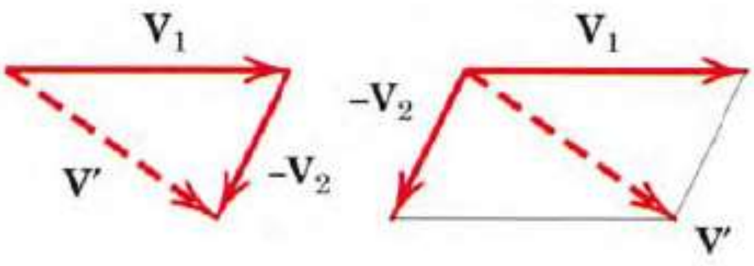


Figure 1/3

Any two or more vectors whose sum equals a certain vector $\mathbf{V}$ are said to be the *components* of that vector. Thus, the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ in Fig. 1/4*a* are the components of $\mathbf{V}$ in the directions 1 and 2, respectively. It is usually most convenient to deal with vector components which are mutually perpendicular; these are called *rectangular components*. The vectors $\mathbf{V}_x$ and $\mathbf{V}_y$ in Fig. 1/4*b* are the x- and y-components, respectively, of $\mathbf{V}$. Likewise, in Fig. 1/4*c*, $\mathbf{V}_{x'}$ and $\mathbf{V}_{y'}$ are the x'- and y'-components of $\mathbf{V}$. When expressed in rectangular components, the direction of the vector with respect to, say, the x-axis is clearly specified by the angle θ, where

$$\boxed{\theta = \tan^{-1} \frac{V_y}{V_x}}$$

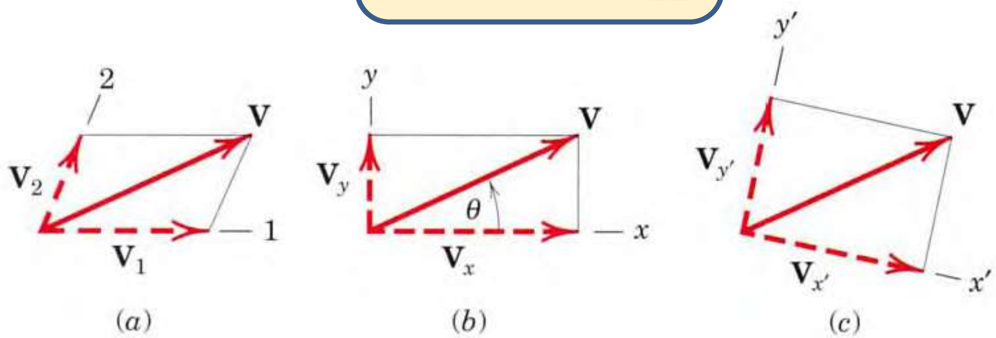


Figure 1/4

A vector $\mathbf{V}$ may be expressed mathematically by multiplying its magnitude V by a vector $\mathbf{n}$ whose magnitude is one and whose direction coincides with that of $\mathbf{V}$. The vector $\mathbf{n}$ is called a *unit vector*. Thus,

$$\boxed{\mathbf{V} = V\mathbf{n}}$$

In this way both the magnitude and direction of the vector are conveniently contained in one mathematical expression. In many problems, particularly three-dimensional ones, it is convenient to express the rectangular components of $\mathbf{V}$, Fig. 1/5, in terms of unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, which are vectors in the x-, y-, and z-directions, respectively, with unit magnitudes. Because the vector $\mathbf{V}$ is the vector sum of the components in the x-, y-, and z-directions, we can express $\mathbf{V}$ as follows:

$$\boxed{\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}}$$

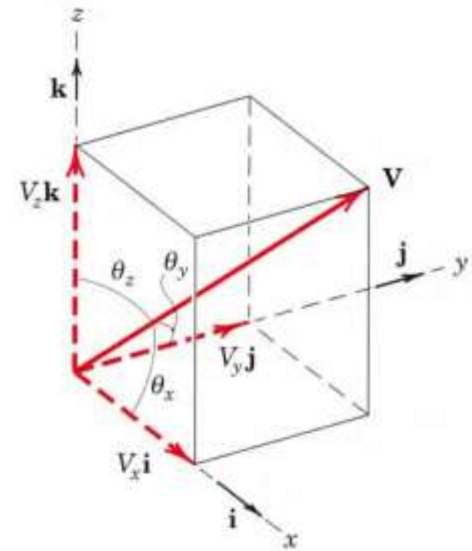


Figure 1/5

We now make use of the *direction cosines* l, m, and n of $\mathbf{V}$, which are defined by

$$\boxed{l = \cos\,\theta_x \qquad m = \cos\,\theta_y \qquad n = \cos\,\theta_z}$$

Thus, we may write the magnitudes of the components of $\mathbf{V}$ as

$$\boxed{V_x = lV \qquad V_y = mV \qquad V_z = nV}$$

where, from the Pythagorean theorem,

$$\boxed{V^2 = V_x{}^2 + V_y{}^2 + V_z{}^2}$$

Note that this relation implies that $l^2 + m^2 + n^2 = 1$.