University of Science and Technology

Faculty of Engineering

first Year – Semester (1)

physics (1)

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Lec (2)

SCALAR AND VECTOR QUANTITIES

Scalar Quantities:

Scalar quantities are those quantities which are having only magnitude but no direction.

Examples:

Mass, length, density, volume, energy, temperature, electric charge, current, electric potential etc.

Vector Quantities:

Vector quantities are those quantities which are having both magnitude as well as direction.

Examples:

Displacement, velocity, acceleration, force, electric intensity, magnetic intensity etc.

Representation of Vector:

A vector is represented by a straight line with an arrow head. Here, the length of the line represents the magnitude and arrow head gives the direction of vector.

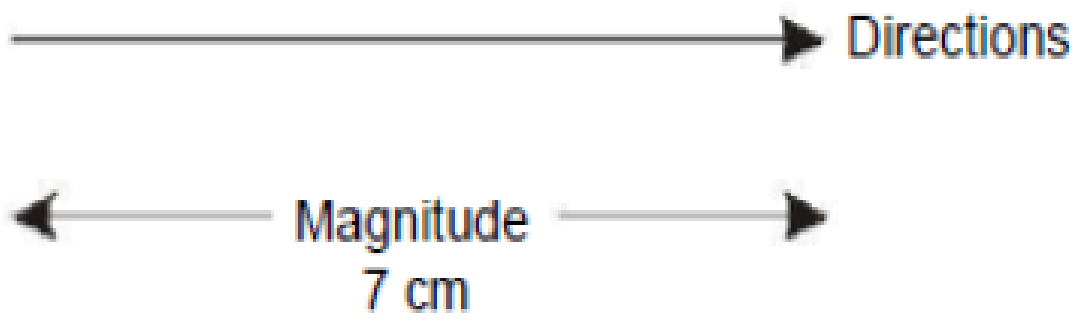


Figure:2.1

Types of Vectors

Negative Vectors:

The negative of a vector is defined as another vector having same magnitude but opposite in direction.

i.e., any vector $\hat{A}$ and its negative vector [$-\hat{A}$] are as shown.

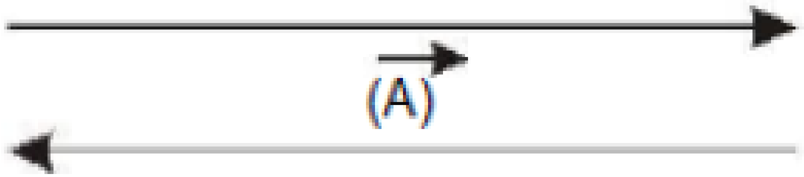


Figure:2.2

Equal Vector:

Two or more vectors are said to be equal, if they have same magnitude and direction.

If $\vec{A}$ and $\vec{B}$ are two equal vectors then

$$A = B$$

Figure:2.3

Unit Vector: A vector divided by its magnitude is called a unit vector. It has a magnitude one unit and direction same as the direction of given vector. It is denoted by $\hat{A}$.

$$\hat{A} = \frac{\vec{A}}{A}$$

Collinear Vectors:

Two or more vectors having equal or unequal magnitudes, but having same direction are called collinear vectors

Figure:2.4

Zero Vector:

A vector having zero magnitude and arbitrary direction (be not fixed) is called zero vector. It is denoted by O.

Addition of Vectors

(i) Triangle law of vector addition.

If two vectors can be represented in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented in magnitude and direction, by third side of the triangle taken in the opposite order (Fig. 2.5).

Magnitude of the resultant is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

And direction of the resultant is given by

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

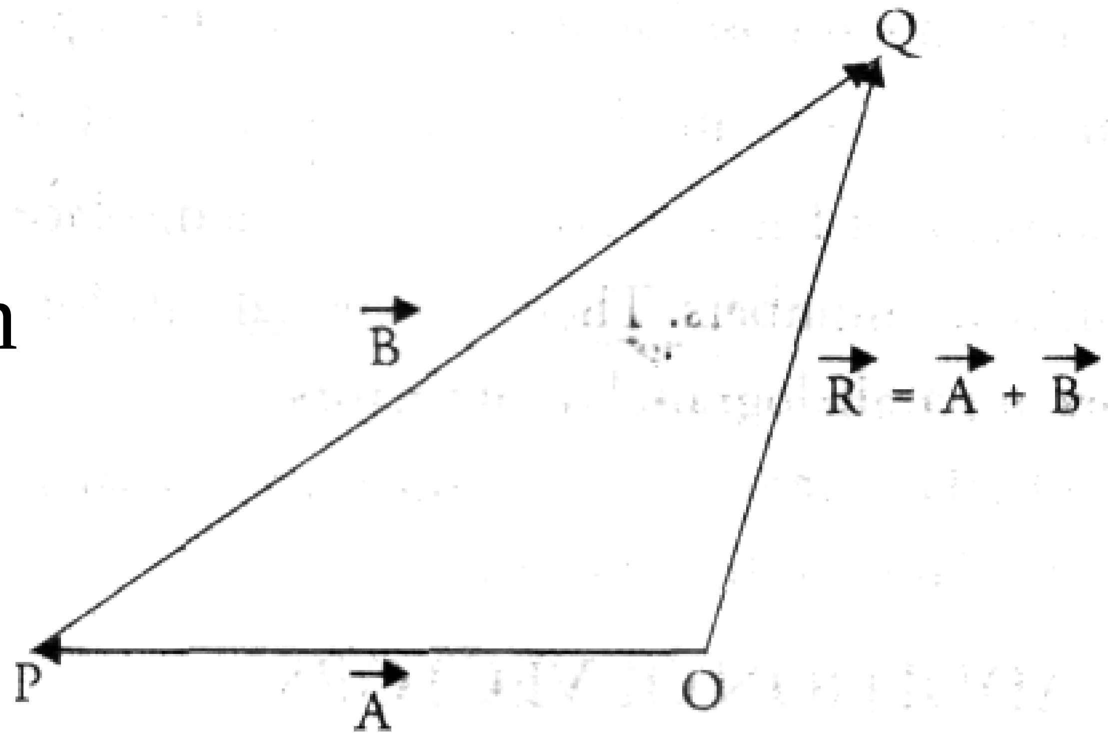


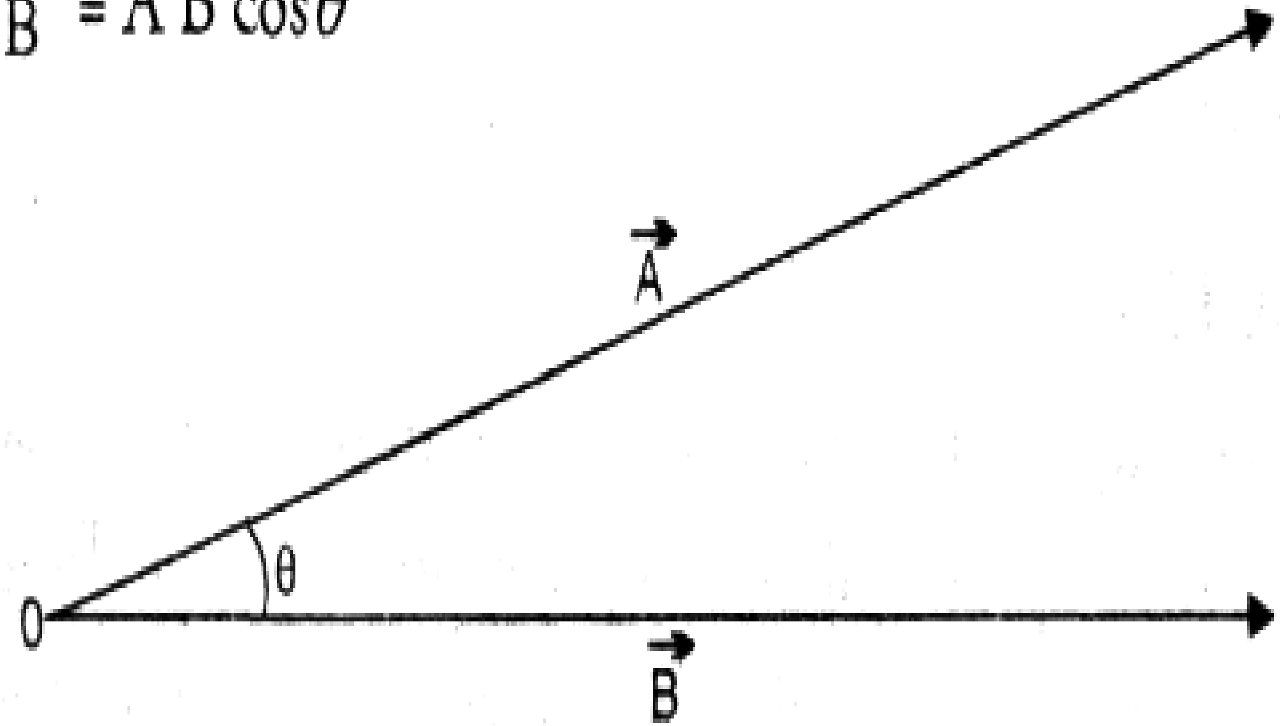
Figure:2.5

2.3 SCALAR AND VECTOR PRODUCT

Multiplication of Vectors

(i) Scalar (or dot) Product: It is defined as the product of magnitude of two vectors and the cosine of the smaller angle between them. The resultant is scalar. The dot product of vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A}.\vec{B} = A\,B\cos\theta$$



(ii) Vector (or Cross) Product: It is defined as a vector having a magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle between them and is in the direction perpendicular to the plane containing the two vectors.

Thus, the vector product of two vectors A and B is equal to

$$\vec{A} \times \vec{B} = AB\, sin\theta\ \hat{n}$$

Example :

Given two vectors, $\vec{\mathbf{A}} = 2\,\hat{\mathbf{i}} + -3\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, find: (a) $\left|\vec{\mathbf{A}}\right|$; (b) $\left|\vec{\mathbf{B}}\right|$; (c) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$; (d) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$; (e) a unit vector $\hat{\mathbf{A}}$ pointing in the direction of $\vec{\mathbf{A}}$; (f) a unit vector $\hat{\mathbf{B}}$ pointing in the direction of $\vec{\mathbf{B}}$.

(a) $\left|\vec{\mathbf{A}}\right| = \left(2^2 + (-3)^2 + 7^2\right)^{1/2} = \sqrt{62} = 7.87$. (b) $\left|\vec{\mathbf{B}}\right| = \left(5^2 + 1^2 + 2^2\right)^{1/2} = \sqrt{30} = 5.48$.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\,\hat{\mathbf{i}} + (A_y + B_y)\,\hat{\mathbf{j}} + (A_z + B_z)\,\hat{\mathbf{k}}$$

(c) $$= (2+5)\,\hat{\mathbf{i}} + (-3+1)\,\hat{\mathbf{j}} + (7+2)\,\hat{\mathbf{k}}$$

$$= 7\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} + 9\,\hat{\mathbf{k}}.$$

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = (A_x - B_x)\,\hat{\mathbf{i}} + (A_y - B_y)\,\hat{\mathbf{j}} + (A_z - B_z)\,\hat{\mathbf{k}}$$

(d) $$= (2-5)\,\hat{\mathbf{i}} + (-3-1)\,\hat{\mathbf{j}} + (7-2)\,\hat{\mathbf{k}}$$

$$= -3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}.$$

(e) A unit vector $\hat{\mathbf{A}}$ in the direction of $\vec{\mathbf{A}}$ can be found by dividing the vector $\vec{\mathbf{A}}$ by the magnitude of $\vec{\mathbf{A}}$. Therefore

$$\hat{\mathbf{A}} = \vec{\mathbf{A}} / \left|\vec{\mathbf{A}}\right| = \left(2\,\hat{\mathbf{i}} + -3\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}\right) / \sqrt{62}\,.$$

(f) In a similar fashion, $\hat{\mathbf{B}} = \vec{\mathbf{B}} / \left|\vec{\mathbf{B}}\right| = \left(5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) / \sqrt{30}\,.$

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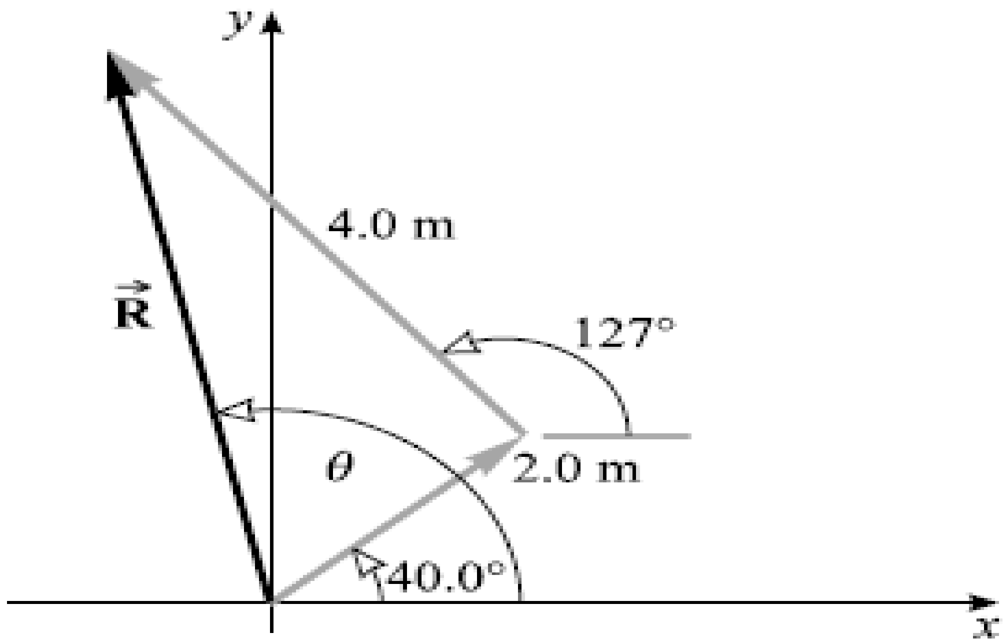
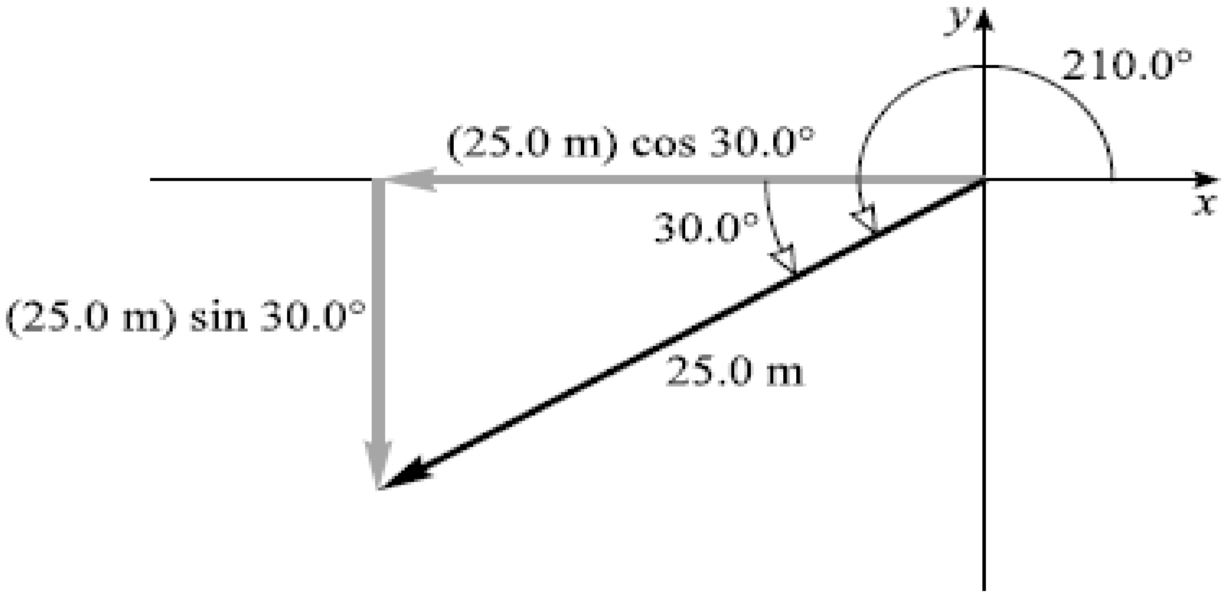
Using the graphical method, find the resultant of the following two displacements: 2.0 m at 40° and 4.0 m at 127°,



Ex : Find the x- and y-components of a 25.0-m displacement at an angle of 210.0°.



The vector displacement and its components are shown in Fig. 1-6. The scalar components are

$$x\text{-component} = -(25.0\text{ m})\cos 30.0° = -21.7\text{ m}$$
$$y\text{-component} = -(25.0\text{ m})\sin 30.0° = -12.5\text{ m}$$

Ex : Three forces that act on a particle are given by $\vec{\mathbf{F}}_1 = (20\hat{\mathbf{i}} - 36\hat{\mathbf{j}} + 73\hat{\mathbf{k}})$ N, $\vec{\mathbf{F}}_2 = (-17\hat{\mathbf{i}} + 21\hat{\mathbf{j}} - 46\hat{\mathbf{k}})$ N, and $\vec{\mathbf{F}}_3 = (-12\hat{\mathbf{k}})$ N. Find their resultant vector. Also find the magnitude of the resultant to two significant figures.

$$R_x = \Sigma F_x = 20\text{ N} - 17\text{ N} + 0\text{ N} = 3\text{ N}$$
$$R_y = \Sigma F_y = -36\text{ N} + 21\text{ N} + 0\text{ N} = -15\text{ N}$$
$$R_z = \Sigma F_z = 73\text{ N} - 46\text{ N} - 12\text{ N} = 15\text{ N}$$

Since $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$, we find

$$\vec{\mathbf{R}} = 3\hat{\mathbf{i}} - 15\hat{\mathbf{j}} + 15\hat{\mathbf{k}}$$

To two significant figures, the three-dimensional pythagorean theorem then gives

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{459} = 21\text{ N}$$