University of Science and Technology

Faculty of Engineering

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physics (1)

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Lec (1)
Units and dimensions

Basic Quantities and Their Dimension

- Dimension has a specific meaning – it denotes the physical nature of a quantity
- Dimensions are denoted with square brackets
  - Length [L]
  - Mass [M]
  - Time [T]

Dimensions and Units

Each dimension can have many actual units. Table below for the dimensions and units of some derived quantities

Dimensions and Units of Four Derived Quantities

Quantity	Area	Volume	Speed	Acceleration
Dimensions	L^2	L^3	L/T	L/T^2
SI units	m^2	m^3	m/s	m/s^2

Dimensional Analysis

- What is "Dimension" ?

  Dimension of a physical quantity is an algebraic combination of L, T and M from which the quantity is found.

- Many physical quantities can be expressed in terms of a combination of fundamental dimensions such as

Length	L
Time	T
Mass	M

- There are physical quantities which are dimensionless:
  - numerical value
  - ratio between the same quantity
  - angle
  - some of the known constants like ln, log, p and etc.

Dimensional Analysis

Dimension analysis can be used to:

a) Check whether an equation is dimensionally correct.

However, dimensionally correct doesn't necessarily mean the equation is correct.

b) Derive an equation.

c) Find out dimension or units of derived quantities.

- Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is [L]

  $[\text{Height}] \rightarrow [L]$

  $[\text{Width}] \rightarrow [L]$

  $[\text{radius}] \rightarrow [L]$

  $[\text{displacement}] \rightarrow [L]$

  here [Height] can be read as "Dimension of Height"

- Area = Length × Width

  So, dimension of area is [Area] = [Length] × [Width]

  $= [L] \times [L] = [L^2]$

  For circle

  Area = πr^2

  $[\text{Area}] = [\pi]\,[r^2]$

  $= [1]\,[L^2]$

  $= [L^2]$

- $[Volume] = [Length] \times [Width] \times [Height] = L \times L \times L = [L^3]$
  For sphere

  $$Volume = \frac{4}{3}\pi r^3$$

  $$[Volume] = \left[\frac{4}{3}\pi\right][r^3] = (1)[L^3] = [L^3]$$

- $$Velocity\ (v) = \frac{displacement}{time}$$

  $$[v] = \frac{[Displacement]}{[time]} = \frac{L}{T} = [M^0L^1T^{-1}]$$

- $$Acceleration\ (a) = \frac{dv}{dt}$$

  $$[a] = \frac{dv \rightarrow \text{kind of velocity}}{dt \rightarrow \text{kind of time}} = \frac{LT^{-1}}{T} = LT^{-2}$$

- Force (F) = ma

  $[F] = [m]\,[a]$

  $= [M]\,[LT^{-2}]$

- Work or Energy = force × displacement

  $[\text{Work}] = [\text{force}]\,[\text{displacement}]$

  $= [M^1L^1T^{-2}]\,[L]$

  $= [M^1L^2T^{-2}]$

- $\text{Pressure} = \dfrac{\text{Force}}{\text{Area}}$

  $$[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$$

Dimensional Analysis

- This is a very important tool to check your work
  - It's also very easy!

- Example:

Doing a problem you get the answer for distance $d = v\,t^2$ (velocity x time^2)

Quantity on left side = L
Quantity on right side = $L / T \times T^2 = L \times T$

- Left units and right units don't match, so answer must be wrong !!

Dimensional Analysis

One can use dimensions only to check the validity of ones expression:

e.g: speed $[v]=[L/T]= [L][T^{-1}]$

Distance (L) traveled by a car at speed V in time T:

$L=V\times T=[L/T]\times[T]=L$

More general expression of dimensional analysis using exponents;

e.g. $[v]=[L^nT^m] \Longrightarrow \frac{L}{T}=[L^nT^m]$ where n=1 and m=-1

Dimensional Analysis

Example:

- The period *P* of a swinging pendulum depends only on the length of the pendulum *d* and the acceleration of gravity *g*.
  - Which of the following formulas for *P* could be correct ?

$$(a)\ P = 2\pi(dg)^2 \qquad (b)\ P = 2\pi\frac{d}{g} \qquad (c)\ P = 2\pi\sqrt{\frac{d}{g}}$$

Given: d has units of length (L) and g has units of (L / T^2).

Dimensional Analysis

Example continue...

Realize that the left hand side P has units of time (T)

- Try the first equation

(a) $P = 2\pi(dg)^2$ $\Rightarrow$ $\left(L \cdot \frac{L}{T^2}\right)^2 = \frac{L^4}{T^4} \neq T$ Not Right !!

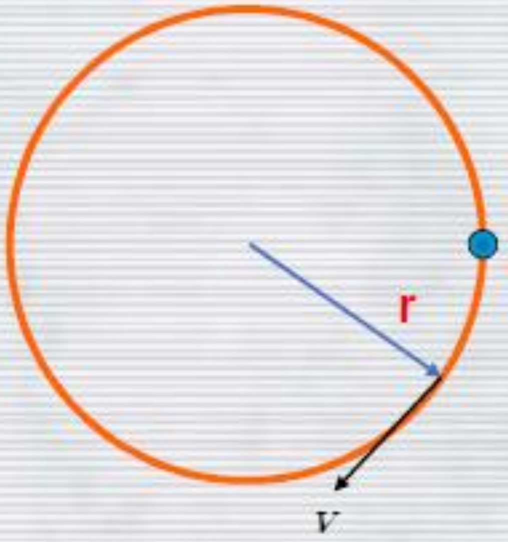
(b) $P = 2\pi \frac{d}{g}$ $\Rightarrow$ $\frac{L}{\frac{L}{T^2}} = T^2 \neq T$ Not Right !!

(c) $P = 2\pi \sqrt{\frac{d}{g}}$ $\Rightarrow$ $\sqrt{\frac{L}{\frac{L}{T^2}}} = \sqrt{T^2} = T$ This has the correct units!!

This must be the answer!!

Dimensional Analysis

Suppose the acceleration a of a circularly moving particle with speed v and radius r is proportional to r^n and v^m. What are n and m?



$$a = kr^n v^m$$

Dimensionless constant

Length

Speed

$$L^1T^{-2} = (L)^n\left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$-m = -2 \Rightarrow m = 2$$

$$n + m = n + 2 = 1 \Rightarrow n = -1$$

$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Dimensional Analysis

Example

The period P of a simple pendulum is the time for one complete swing. How does P depend on the mass m of the bob, the length l of the string, and the acceleration due to gravity g?

We begin by expressing the period P in terms of the other quantities as follows:

$$P = k\, m^x\, l^y\, g^z$$

where k is a constant and x, y, and z are to be determined.

$$P = (M^x)(L^y)(L^z T^{-2z}) \qquad P = (M^x)(L^{y+z})(T^{-2z})$$

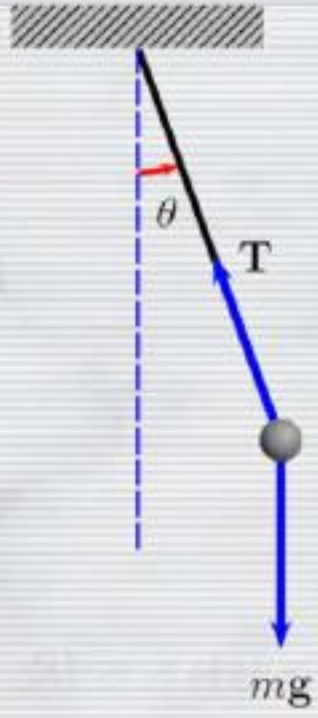
T: $1 = -2z$

M: $0 = x$

L: $0 = y + z$

These equations are easily solved and yield $x = 0$, $z = -\frac{1}{2}$, $y = \frac{1}{2}$

$$P = k\sqrt{\frac{l}{g}}$$



Dimensional Analysis

- The force (F) to keep an object moving in a circle can be described in terms of the velocity (v, dimension L/T) of the object, its mass (m, dimension M), and the radius of the circle (R, dimension L).
  - Which of the following formulas for *F* *could* be correct ?

(a) $F = mvR$ (b) $F=m(v/R)^2$ (c) $F=mv^2/R$

Remember: ***Force*** **has dimensions of ML/T^2**

- There is a famous Einstein's equation connecting energy and mass (relativistic). Using dimensional analysis find which is the correct form of this equation :

(a) $E=mc$ (b) $E=mc^2$ (c) $E=mc^3$

Pervious exam

The acceleration a of a particle moving with uniform speed v in a circle of radius r is given by the expression $a = k\, r^n\, v^m$ (k is dimensionless). Using the dimensional analysis, the values of n and m respectively are:

(a) 1, -2 (b) -1, 2 (c) 1, 2 (d) 2, 3 (e) -2, 3

From Hooks law , F=-kx , where F is the force with dimension of (MLT^{-2}), and x is spring extended length. The dimension of the spring constant k is:

(a) ML^2 (b) ML^2T^2 (c) MT^{-2} (d) $ML^{-2}T^{-2}$ (e) $ML^{-2}T^2$

Quantity	Dimension	MKS unit
Area	L^2	m^2
Volume	L^3	m^3
Velocity	$L \cdot T^{-1}$	$m \cdot s^{-1}$
Acceleration	$L \cdot T^{-2}$	$m \cdot s^{-2}$
Density	$M \cdot L^{-3}$	$kg \cdot m^{-3}$
Momentum	$M \cdot L \cdot T^{-1}$	$kg \cdot m \cdot s^{-1}$
Force	$M \cdot L \cdot T^{-2}$	$kg \cdot m \cdot s^{-2} = \text{newton} = N$
Work, Energy	$M \cdot L^2 \cdot T^{-2}$	$kg \cdot m^2 \cdot s^{-2} = \text{joule} = J$
Power	$M \cdot L^2 \cdot T^{-3}$	$kg \cdot m^2 \cdot s^{-3} = \text{watt} = W$
Pressure	$M \cdot L^{-1} \cdot T^{-2}$	$kg \cdot m^{-1} \cdot s^{-2} = \text{pascal} = Pa$

SOME IMPORTANT ABBREVIATIONS

Symbol	Prefix	Multiplier	Symbol	Prefix	Multiplier
D	Deci	10^{-1}	da	deca	10^{1}
c	centi	10^{-2}	h	hecto	10^{2}
m	milli	10^{-3}	k	kilo	10^{3}
μ	micro	10^{-6}	M	mega	10^{6}
n	nano	10^{-9}	G	giga	10^{9}
P	Pico	10^{-12}	T	tera	10^{12}
f	femto	10^{-15}	P	Pecta	10^{15}
a	atto	10^{-18}	E	exa	10^{18}

EXERCISES

Multiple Choice Questions

1. $[ML^{-1}T^{-2}]$ is the dimensional formula of
   (A) Force
   (B) Coefficient of friction
   (C) Modulus of elasticity
   (D) Energy.

2. 10^5Fermi is equal to
   (A) 1 meter
   (B) 100 micron
   (C) 1 Angstrom
   (D) 1 mm

3. rad / sec is the unit of
   (A) Angular displacement
   (B) Angular velocity
   (C) Angular acceleration
   (D) Angular momentum

4. What is the unit for measuring the amplitude of a sound?

(A) Decibel
(B) Coulomb
(C) Hum
(D) Cycles

5. The displacement of particle moving along x-axis with respect to time is $x=at+bt^2-ct^3$. The dimension of c is
(A) LT^{-2}
(B) T^{-3}
(C) LT^{-3}
(D) T^{-3}