2.1. *Orthographic Projection Fundamentals*

For OP, the horizontal or top view, the front view, and the profile or side view corre- spond to X, Y (Z = 0), X, Z (Y = 0), and Y, Z (X = 0) coordinates, respectively. For example, Figure 1 shows the views of the face with n vertices defined by coordinates (X_n, Y_n, Z_n), $n \in [1, 6]$. Thus, the top view corresponds to the X_n, Y_n, the front view to X_n, Z_n, and the side view to the Y_n, Z_n coordinates.

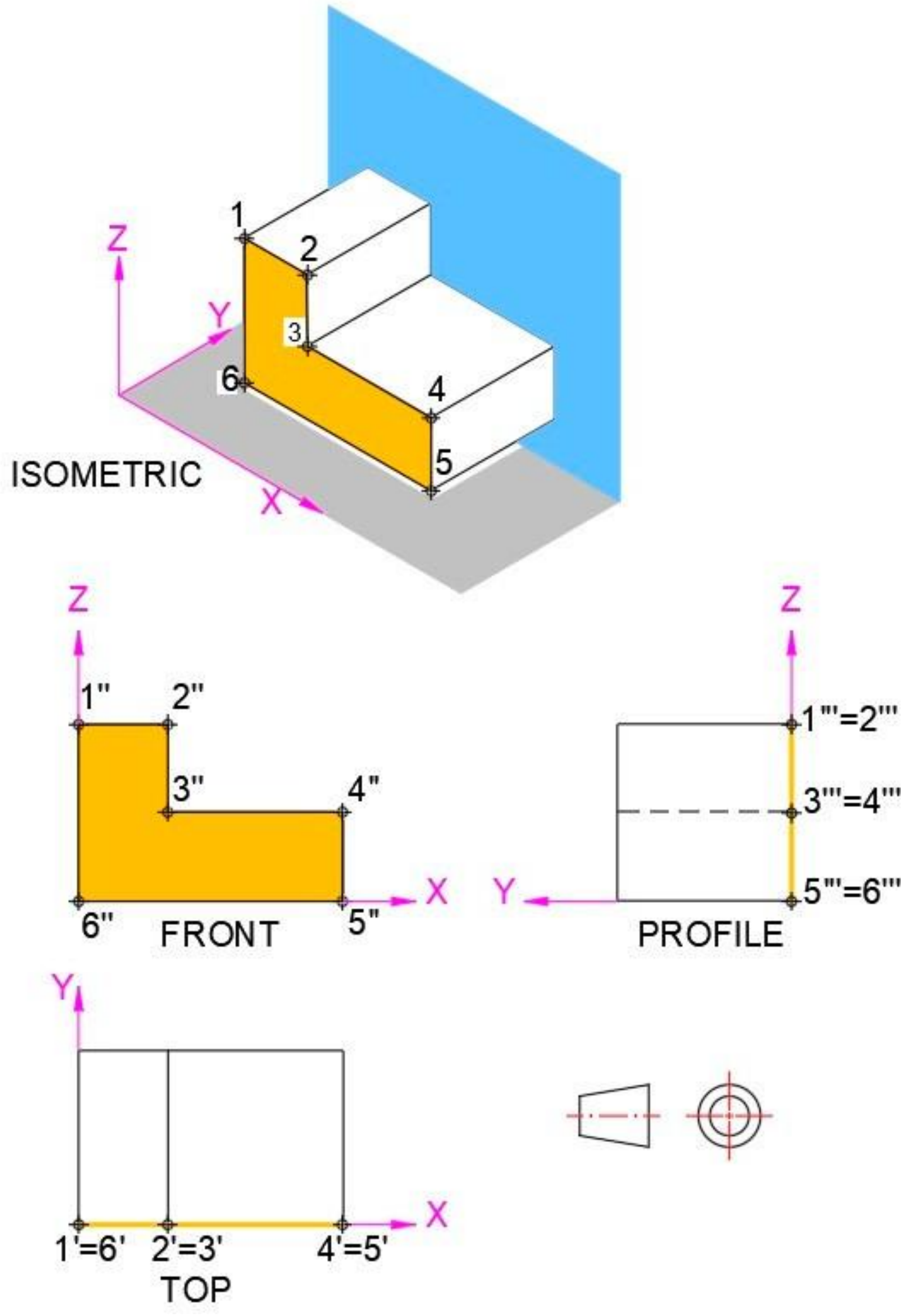

Figure 1. Views obtained from orthographic first-angle projection.

2. *CAD Fundamentals*

The CAD fundamentals used here to solve 2D and 3D DG problems are the following: (i) reference system; (ii) point of view; (iii) change of coordinate system; and (iv) graphic windows.

1. Reference System

Defining the reference system is the first step to obtaining the OP views. This reference system consists of the following components: world coordinate system (WCS), user coordinate system (UCS), reference plane, and coordinate system.

- World and User Coordinate Systems

WCS is a fixed Cartesian coordinate system which is the reference for any other UCS. The latter is a mobile Cartesian coordinate system that establishes the XY reference plane, horizontal and vertical directions, and rotation axes (Figure 2).

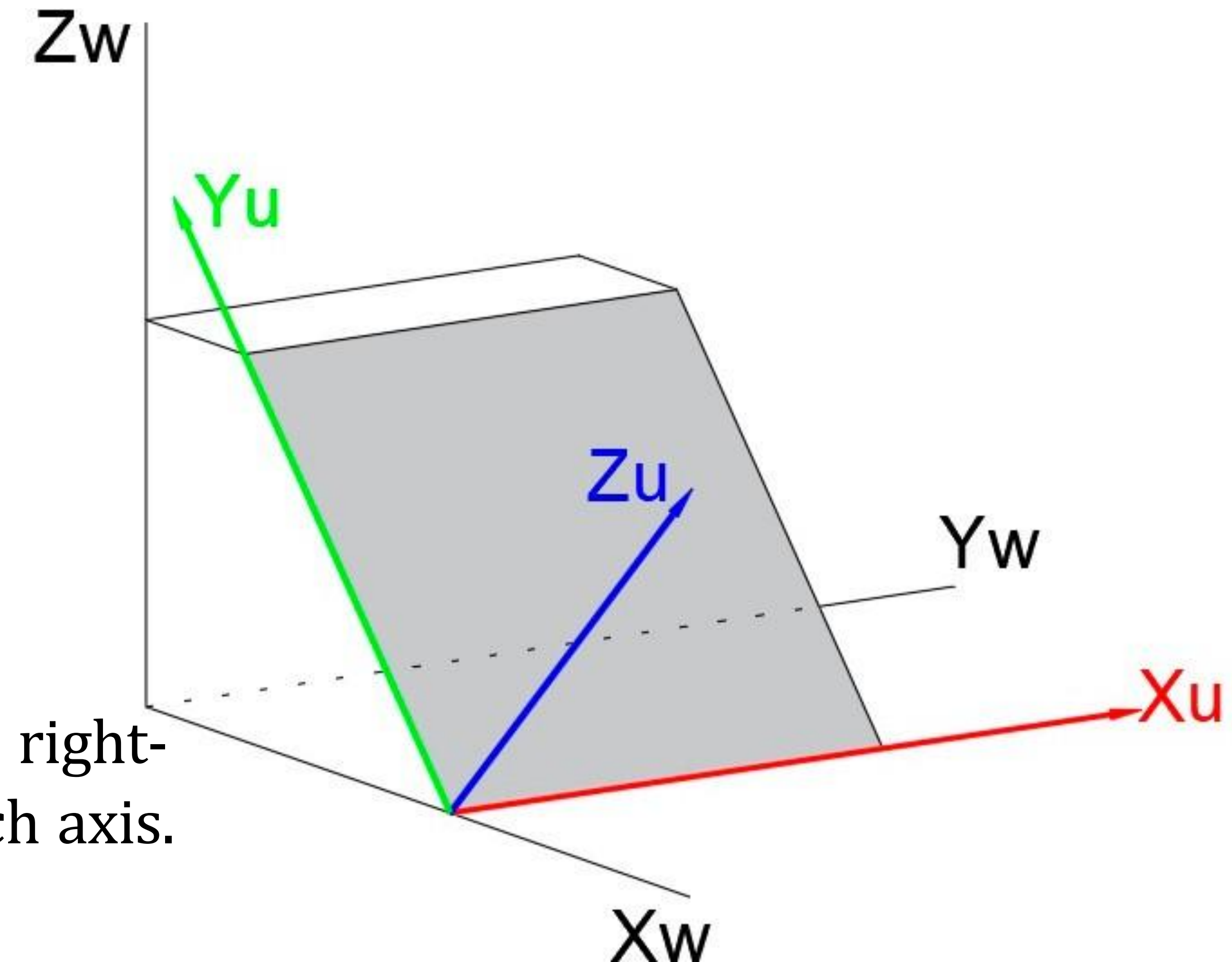

Figure 2. Samples of WCS (Xw, Yw, Zw) and UCS (Xu, Yu, Zu).

Once the reference plane is defined, the Z axis is determined by applying the right-hand rule. This rule is also used to specify the direction of positive rotation about each axis.

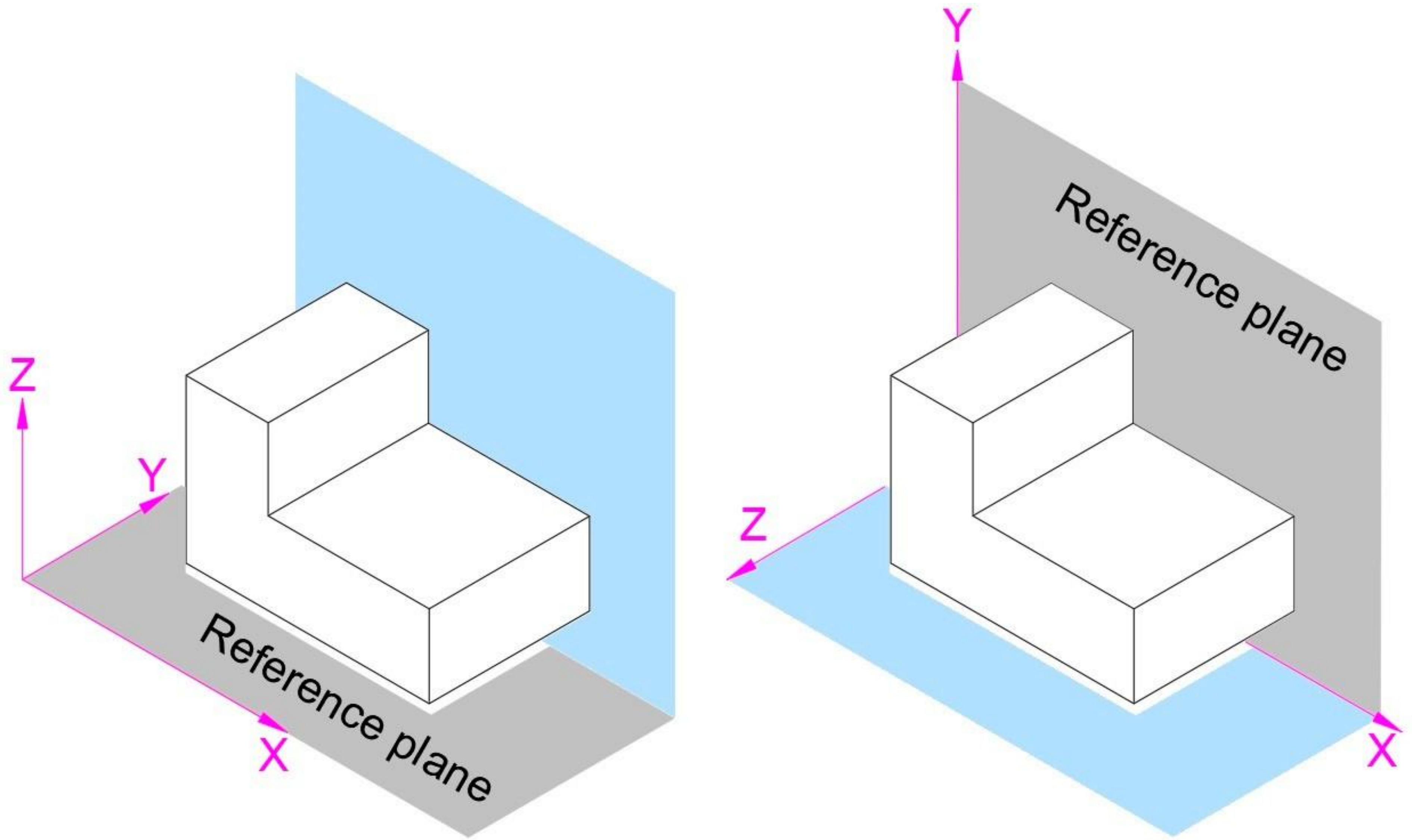

Figure 3. Examples for reference plane positions.

- Coordinate system

The Cartesian coordinate system is used as a reference for the WCS and UCS in CAD. Any point can be located by describing its location with respect to the three axes X, Y, Z. These axes have the coordinate system origin (0, 0, 0) as their intersection point. Figure 4 shows the Cartesian representation of a point P (4, 2, 3), that is, find 4 units on the X axis, 2 on the Y axis, and 3 on the Z axis.

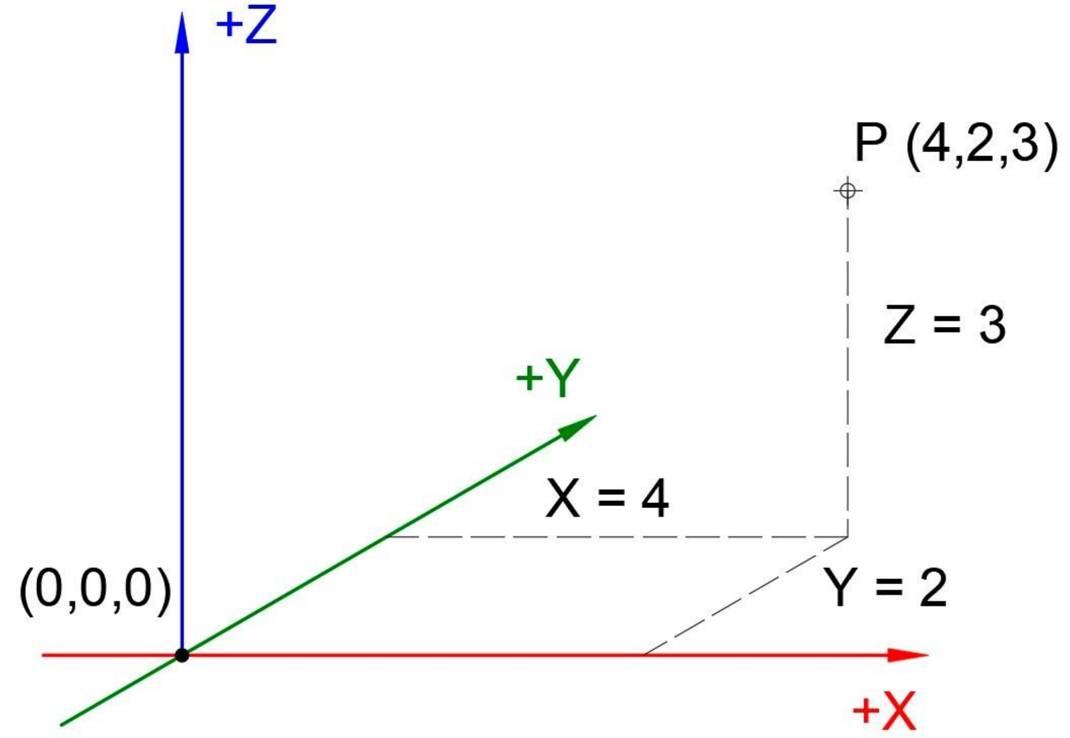

Figure 4. Point P (4, 2, 3) location in the Cartesian coordinate system.

2.2.2. Viewpoint

The point of view defines the line of sight or line of projection of a 3D visualization. The 2D view from a 3D scene is obtained by means of orthographic projection. It is created on a plane perpendicular to the viewing direction (Figure 5).

Figure 6 shows different points of view for the same orientation of the object with respect to the WCS. As can be seen, the projection on the horizontal, vertical, and profile planes does not change, since the WCS remains fixed in both cases.

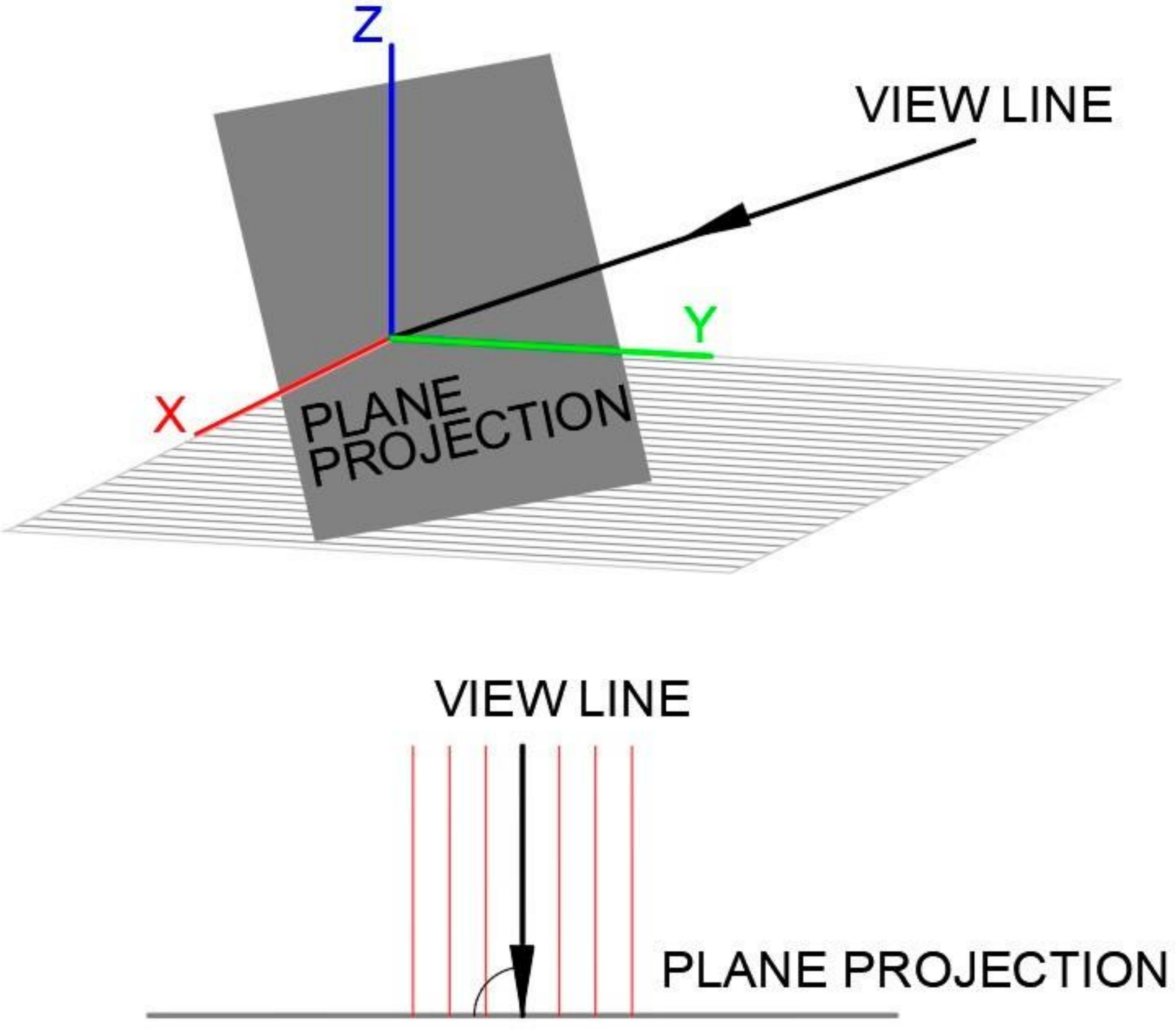

Figure 5. View line and perpendicular projection plane.

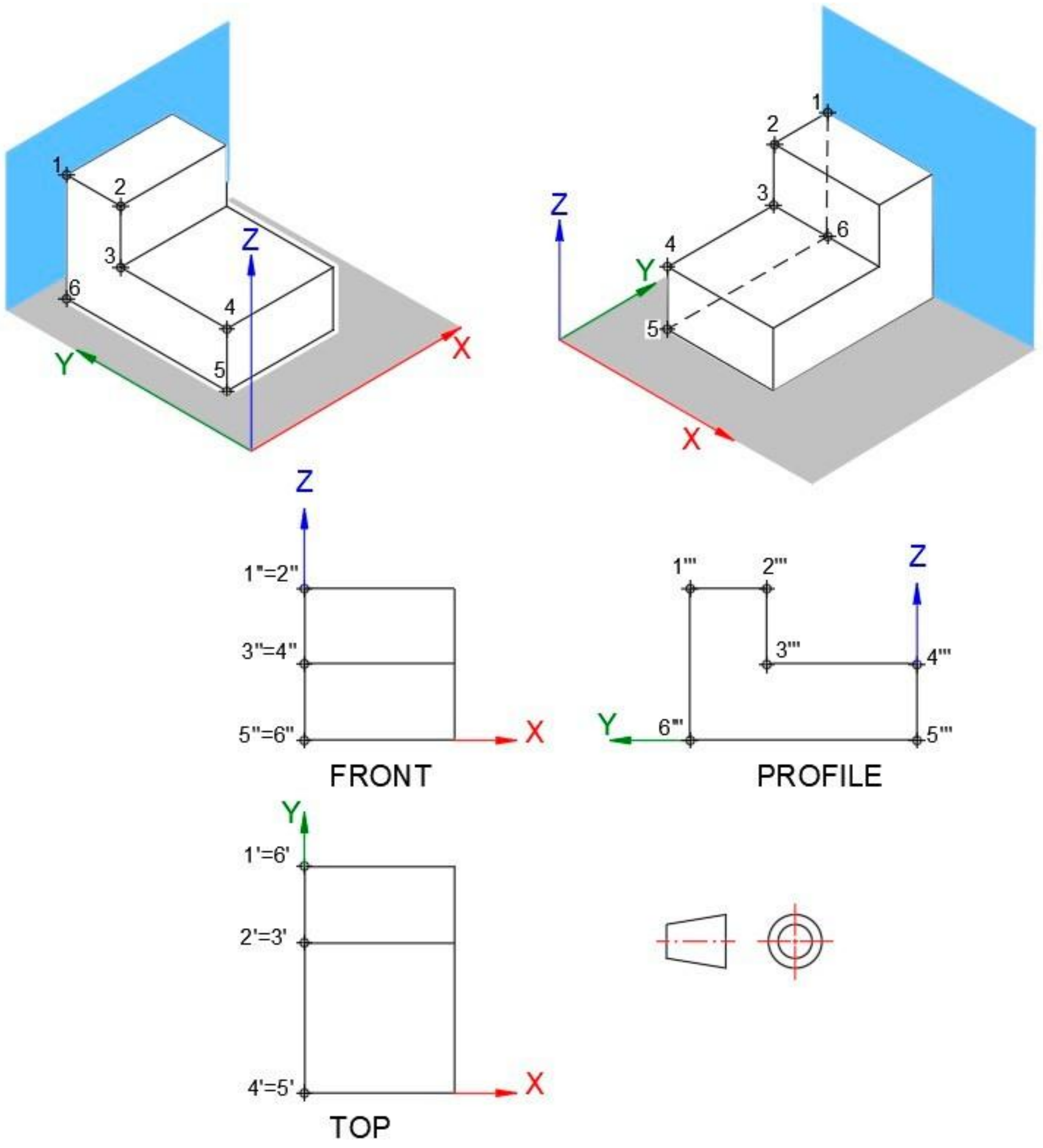

Figure 6. Different points of view of an object and its orthographic projections for the same WCS.

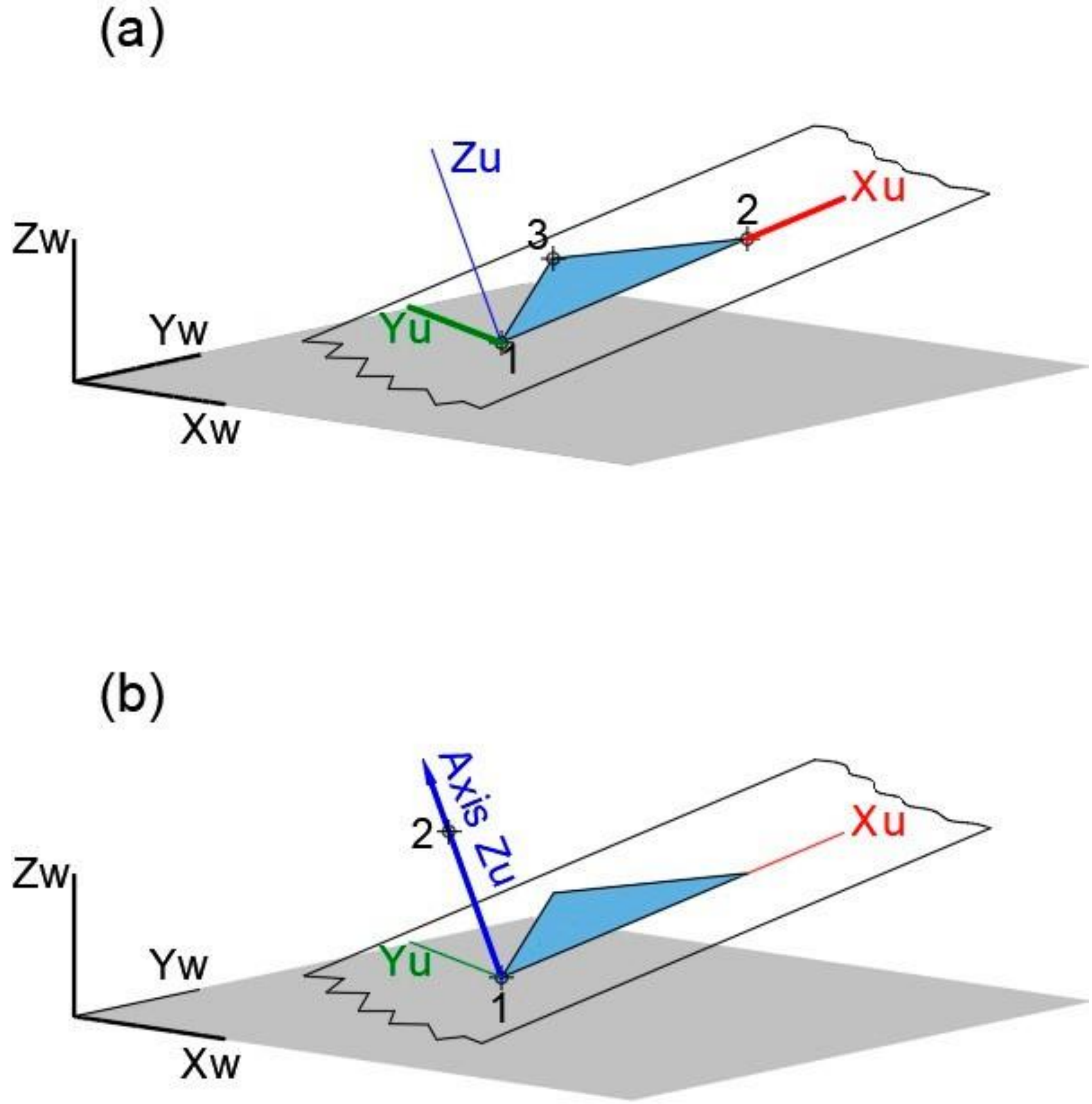

Figure 7. User coordinate system definition: (**a**) XY plane through three points and (**b**) Z axis direction through two points.

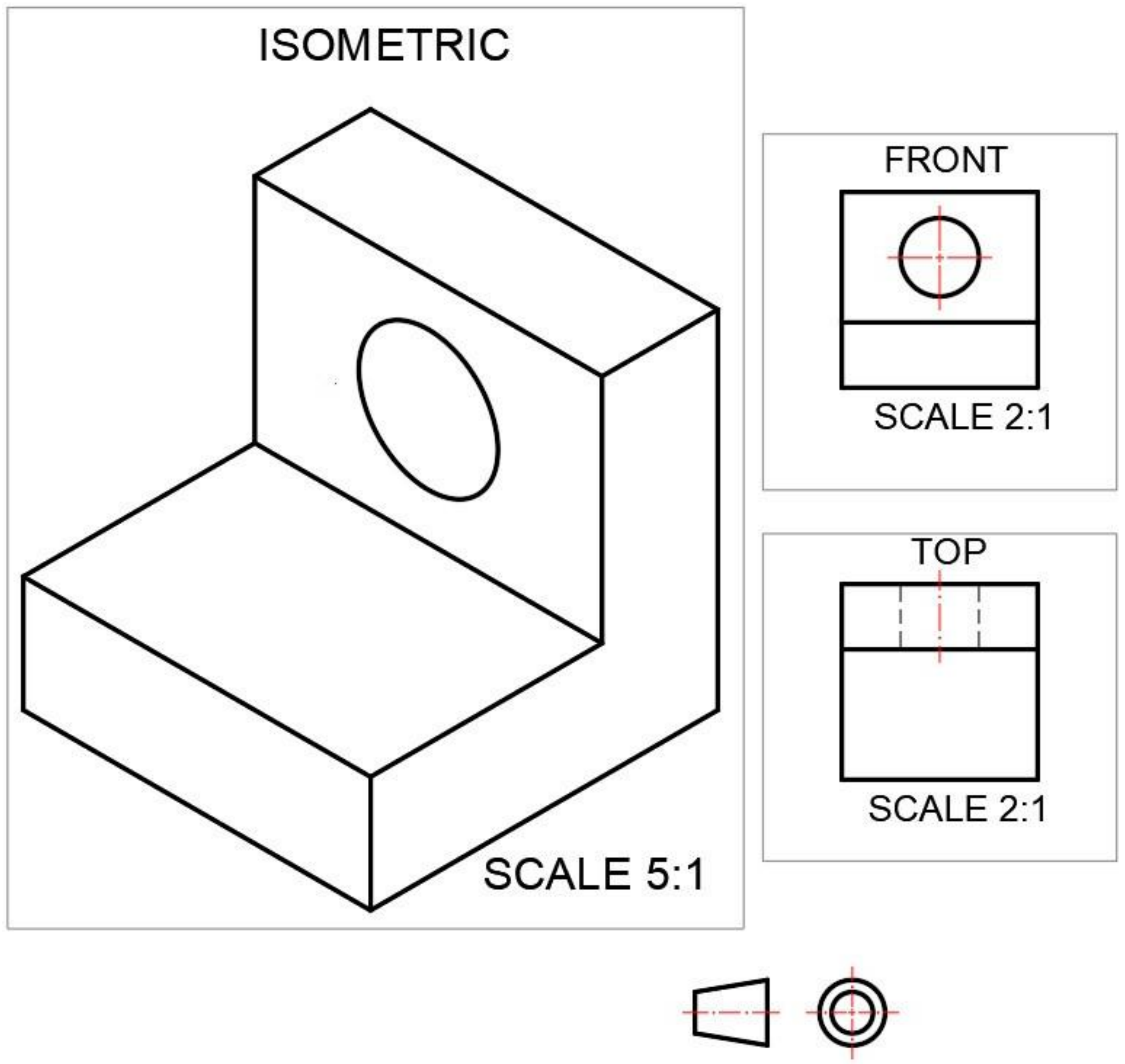

Figure 8. Isometric view at a 5:1 scale, and front and top views at a 2:1 scale of a solid shown in three different viewports.

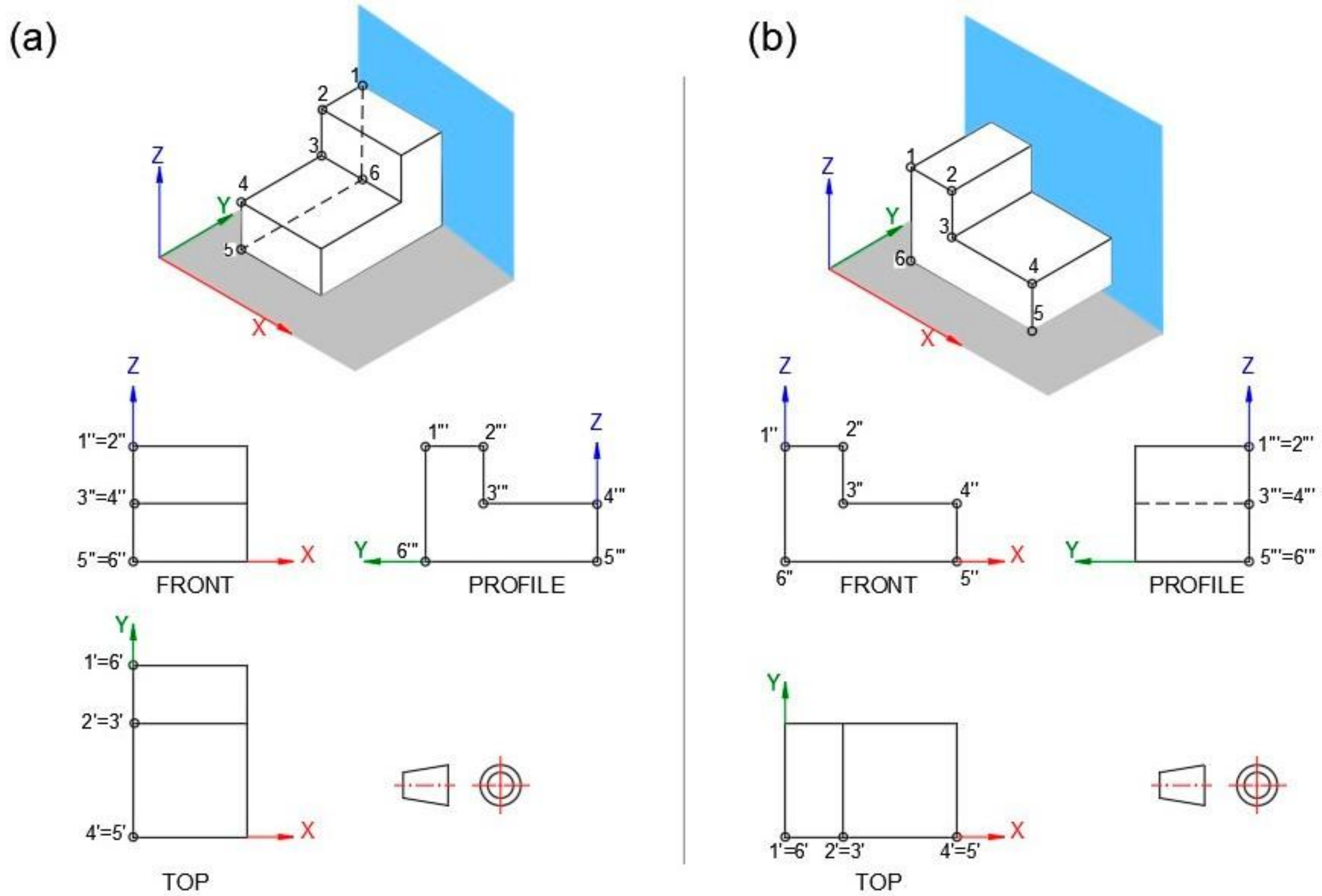

Figure 9. Initial (**a**) and rotated (**b**) positions of a solid for the same point of view.

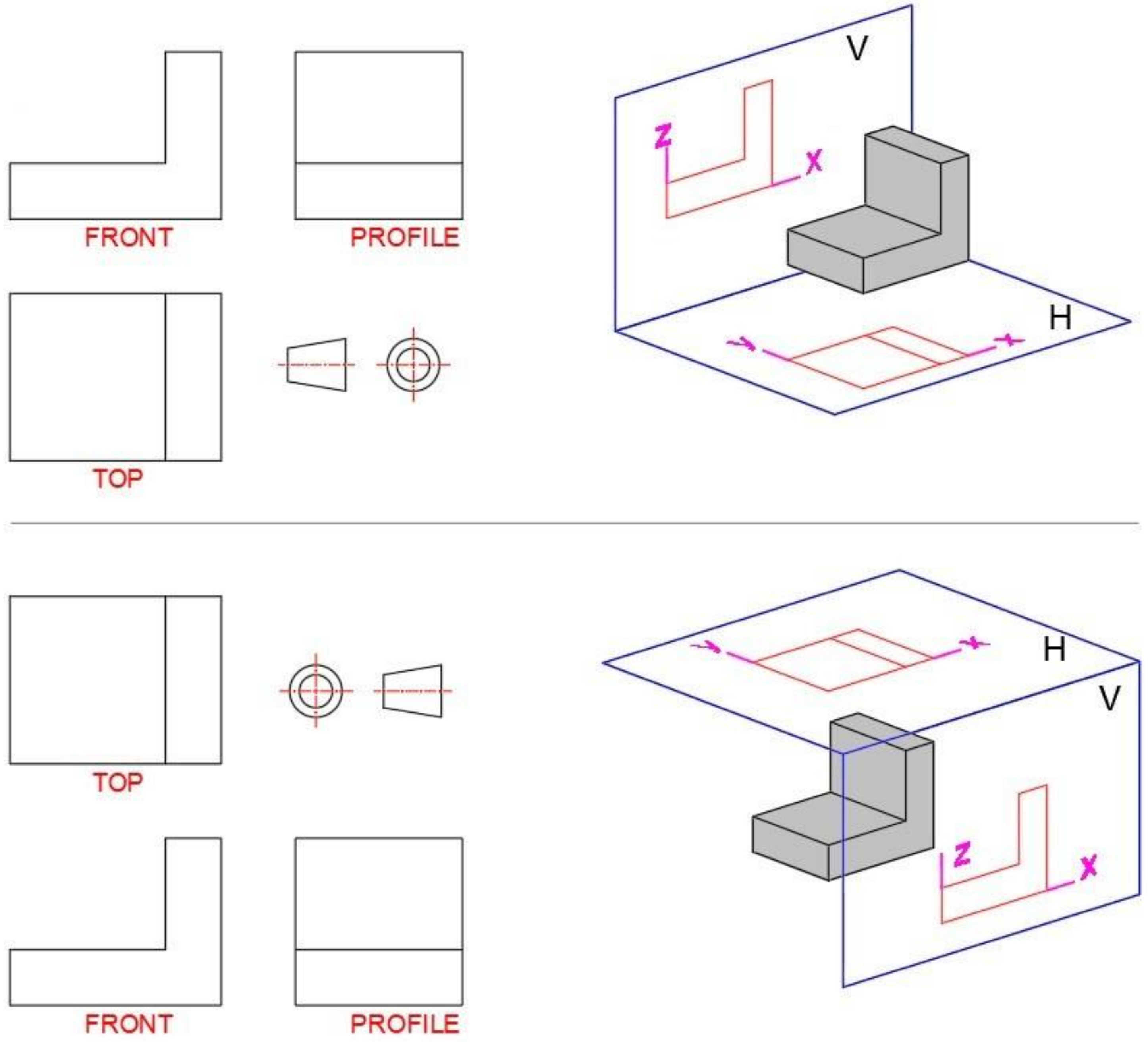

Figure 10. Different conventions for arranging views: first-angle projection (**top**); third-angle projection (**bottom**).

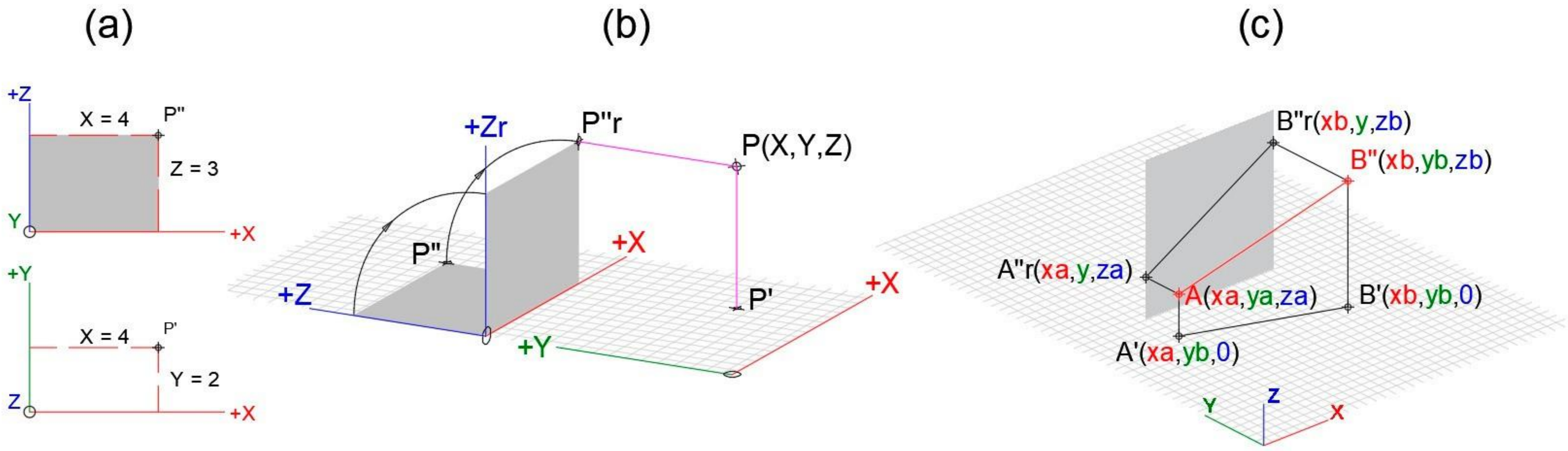

Figure 11. (**a**) Orthographic projections of a point; front view coordinates X, Z and top view coordinates X, Y. (**b**) Front view rotated 90° with respect to the X axis. (**c**) Application to an oblique line defined by points A and B.

3. Results

The application of CADOP is shown in this section to verify the theoretical principles of DG mentioned before.

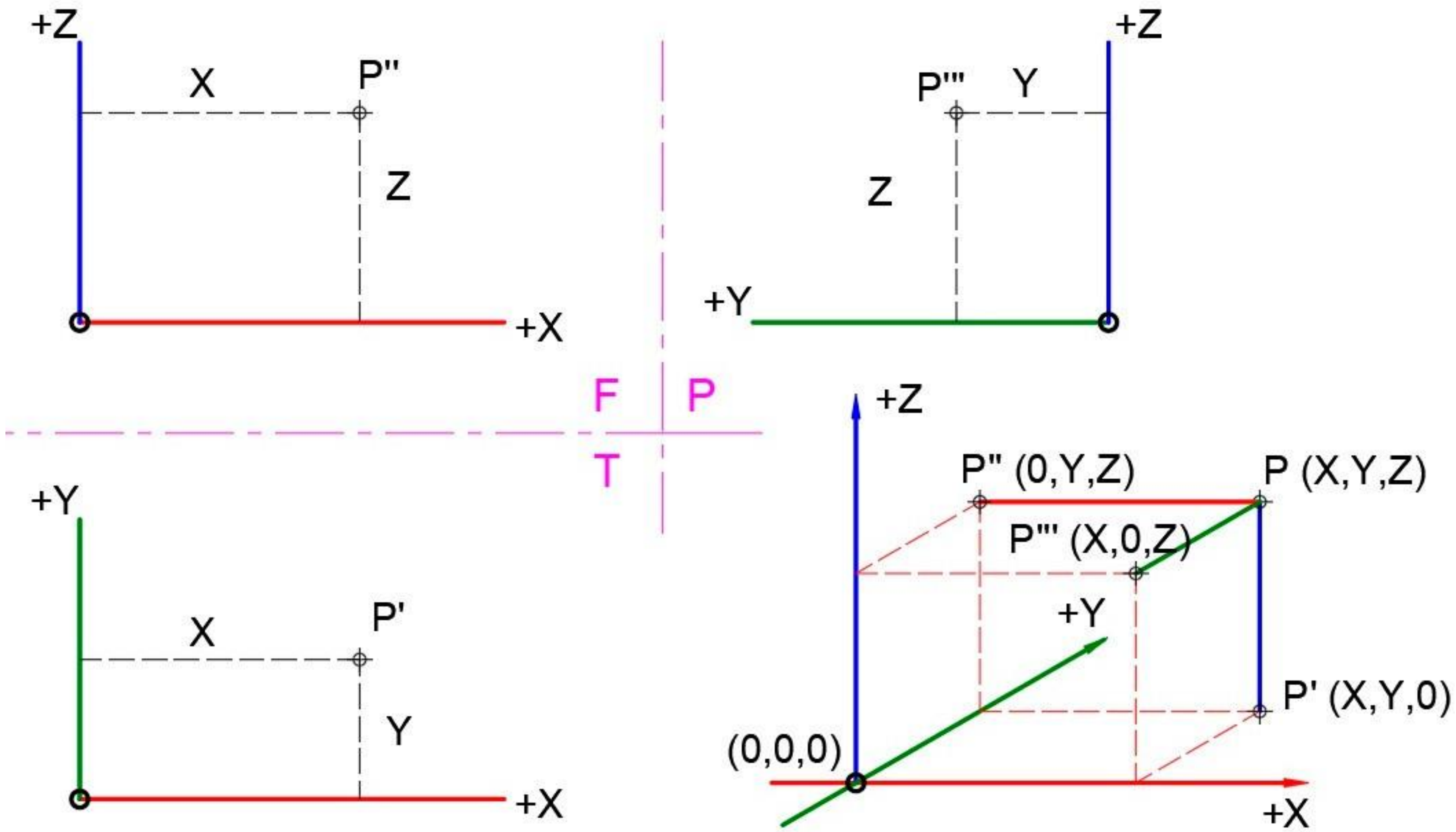

Figure 12. Cartesian coordinates in a multi-view drawing.

The determination of the line projection on an oblique plane with respect to the WCS is another orthogonal view application. The oblique plane must be set as the XY plane of the new UCS. After that, the Zu coordinates of the segment ends are 0. Figure 13 shows the procedure.

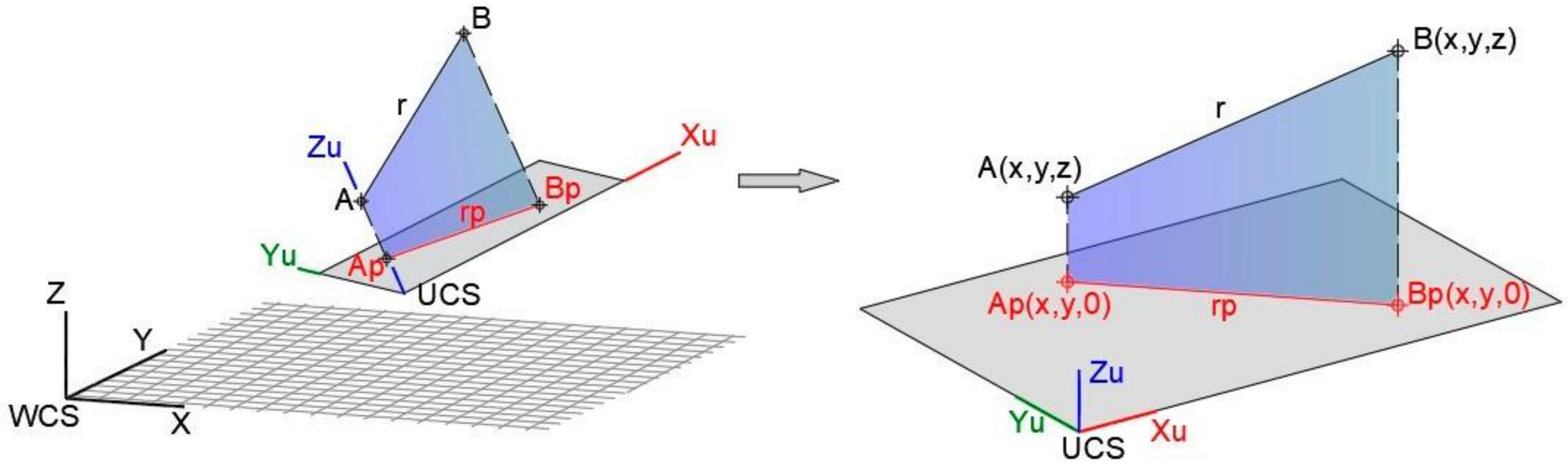

Figure 13. Projection of a line onto an oblique plane.

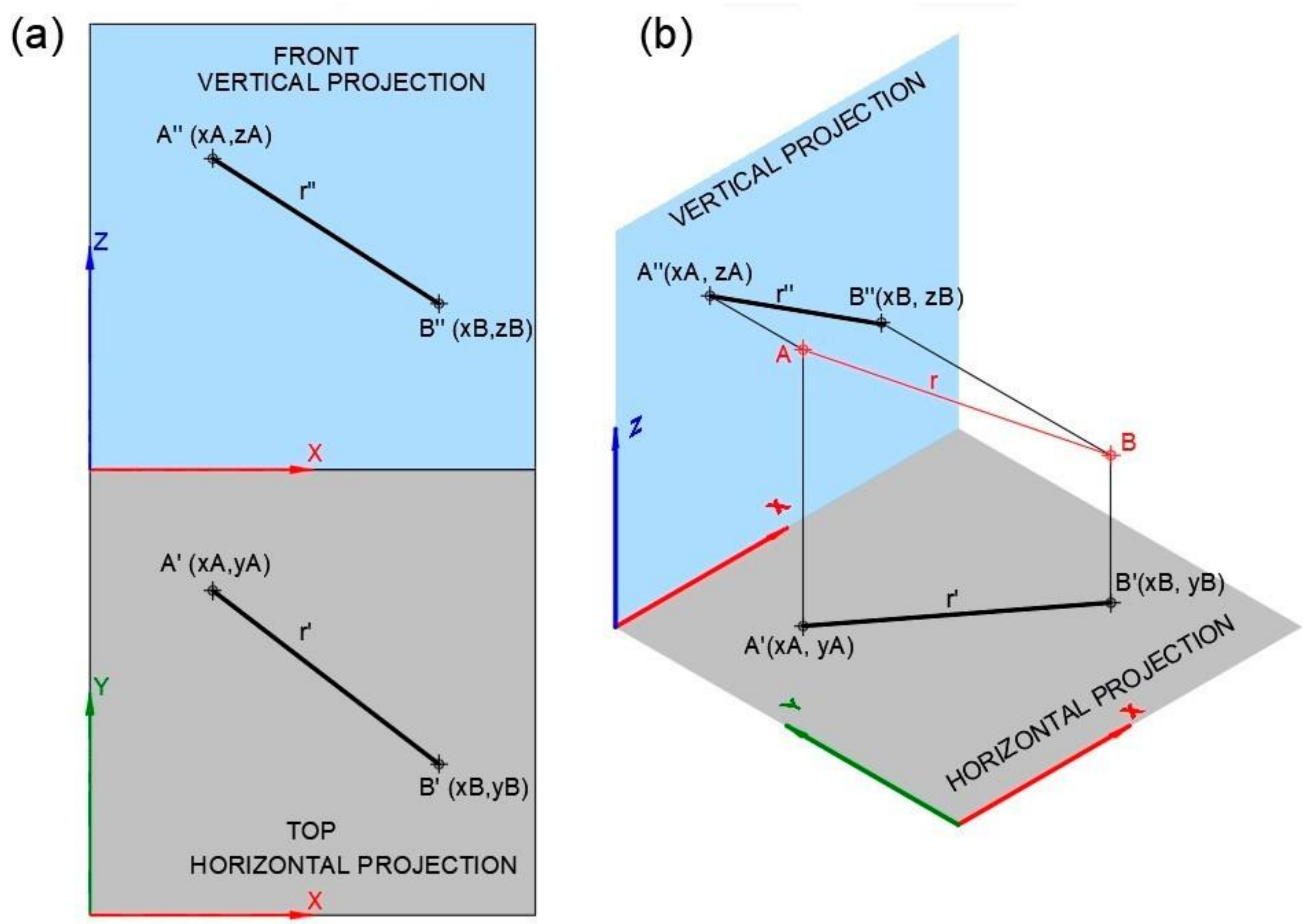

Figure 14. (**a**) Top and front views of line AB. (**b**) Restoring the spatial position of line AB from its horizontal and vertical projections.

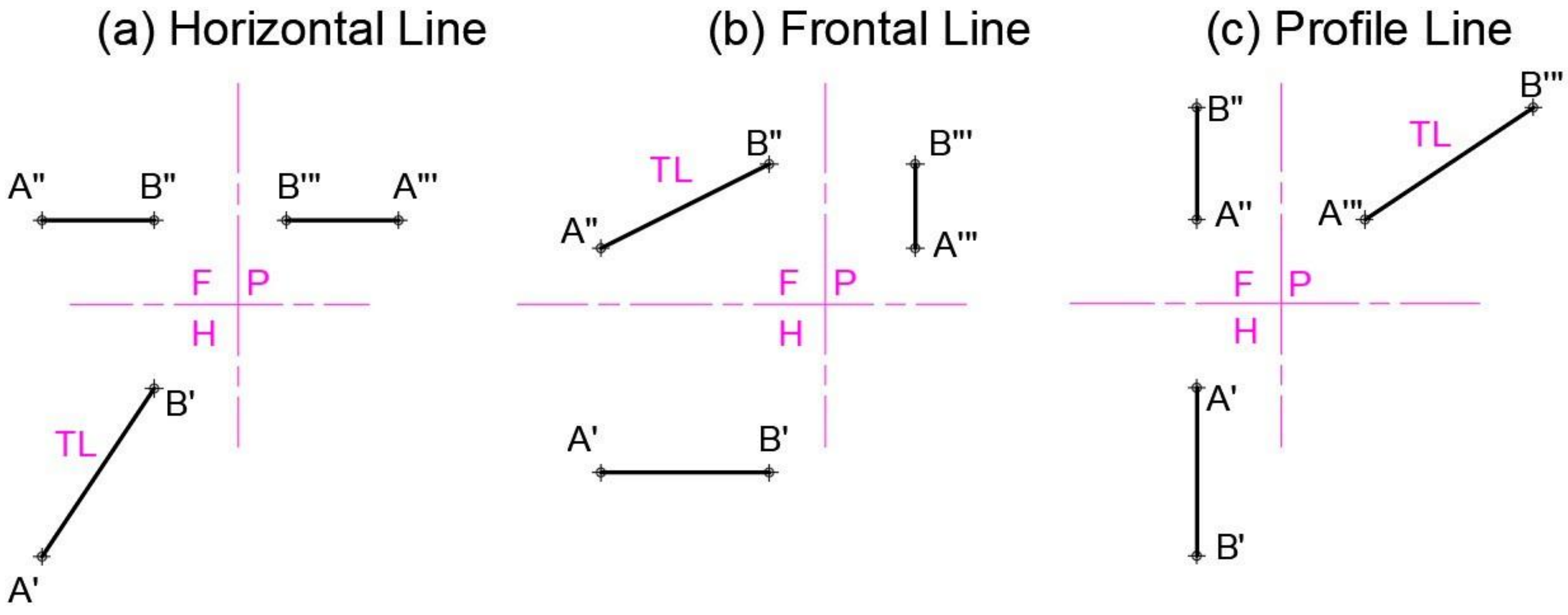

Figure 15. Principal lines: horizontal, frontal, and profile. TL stands for true length.

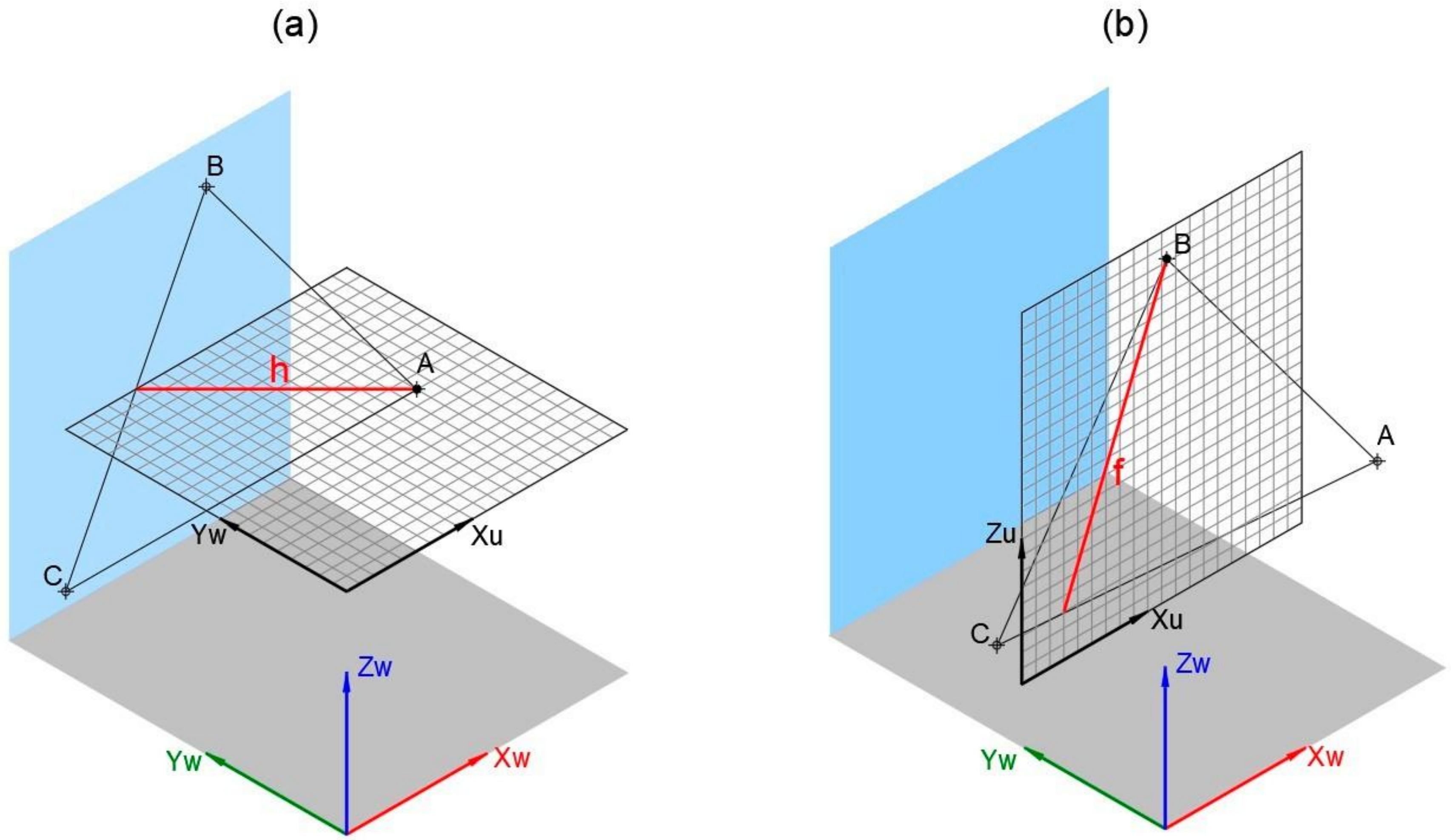

Figure 16. Principal lines through a point of a plane using the intersection of surfaces: (**a**) horizontal line h through point A and (**b**) frontal line f through point B.

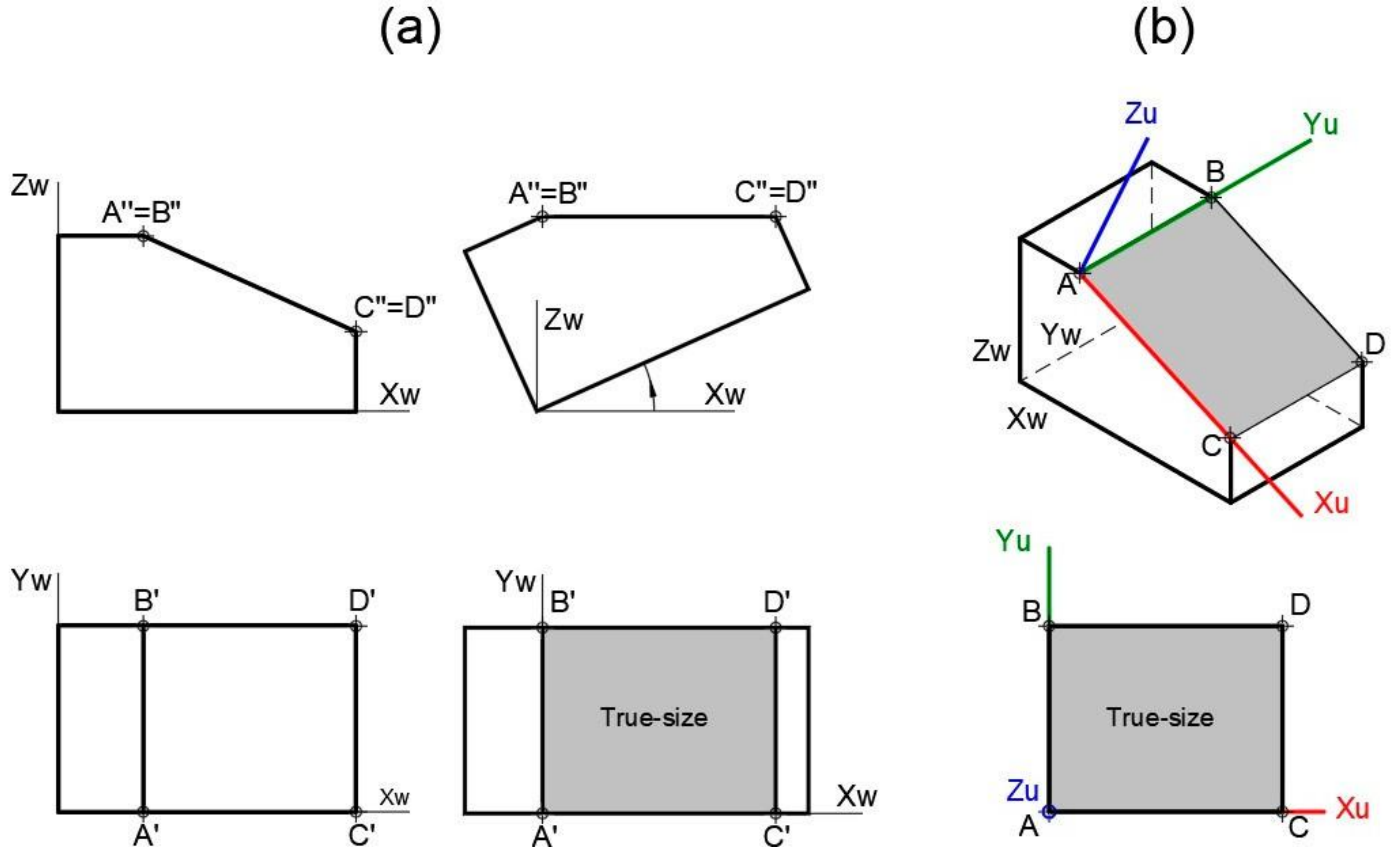

Figure 17. True size of the plane ABCD by (**a**) rotating the body without modifying the coordinate system; and (**b**) considering an UCS.

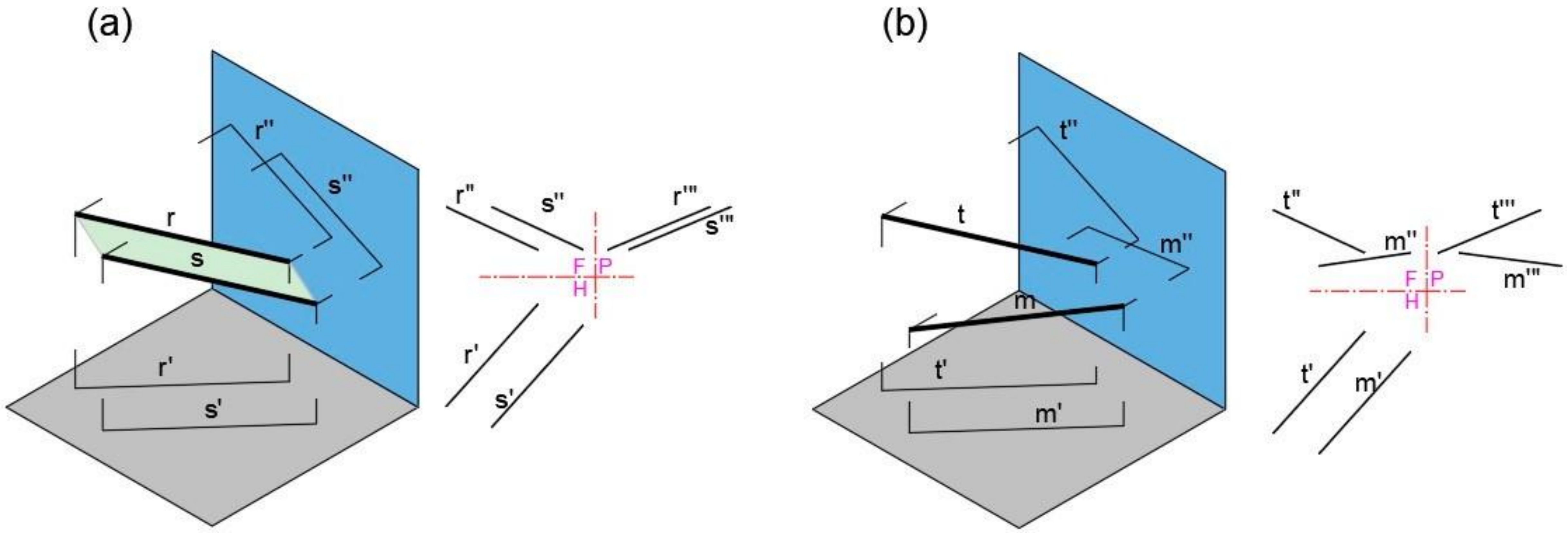

Figure 18. (**a**) Lines *r* and *s* are parallel. (**b**) Lines *t* and *m* are not parallel.

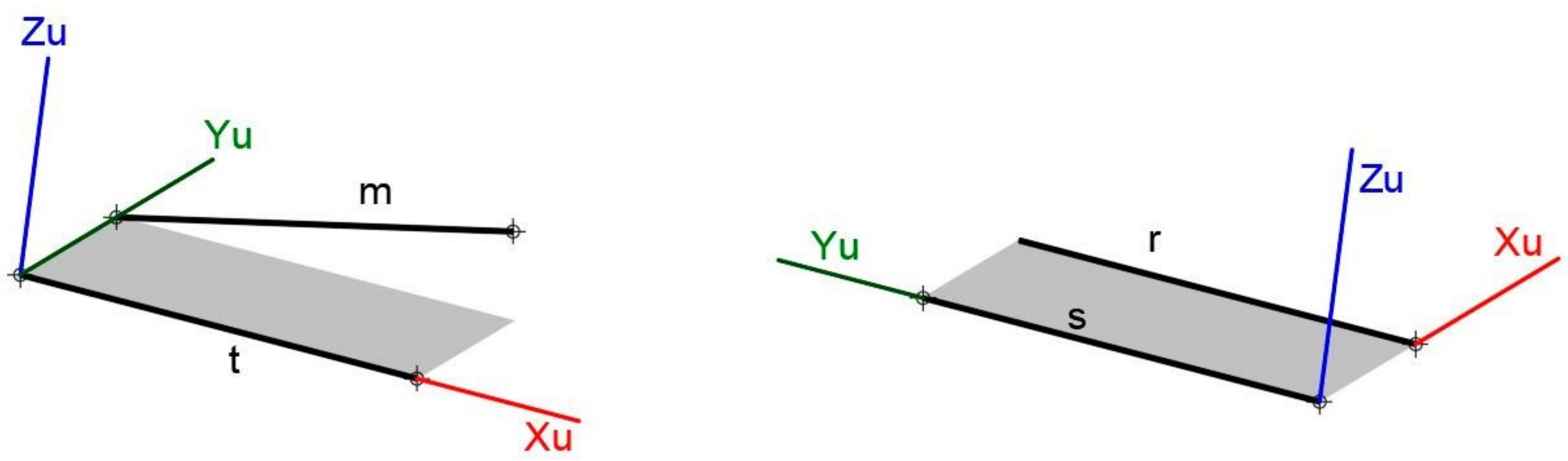

Figure 19. Using the UCS for checking parallelism between lines. Lines *m* and *t* are not parallel, while lines *r* and *s* are.

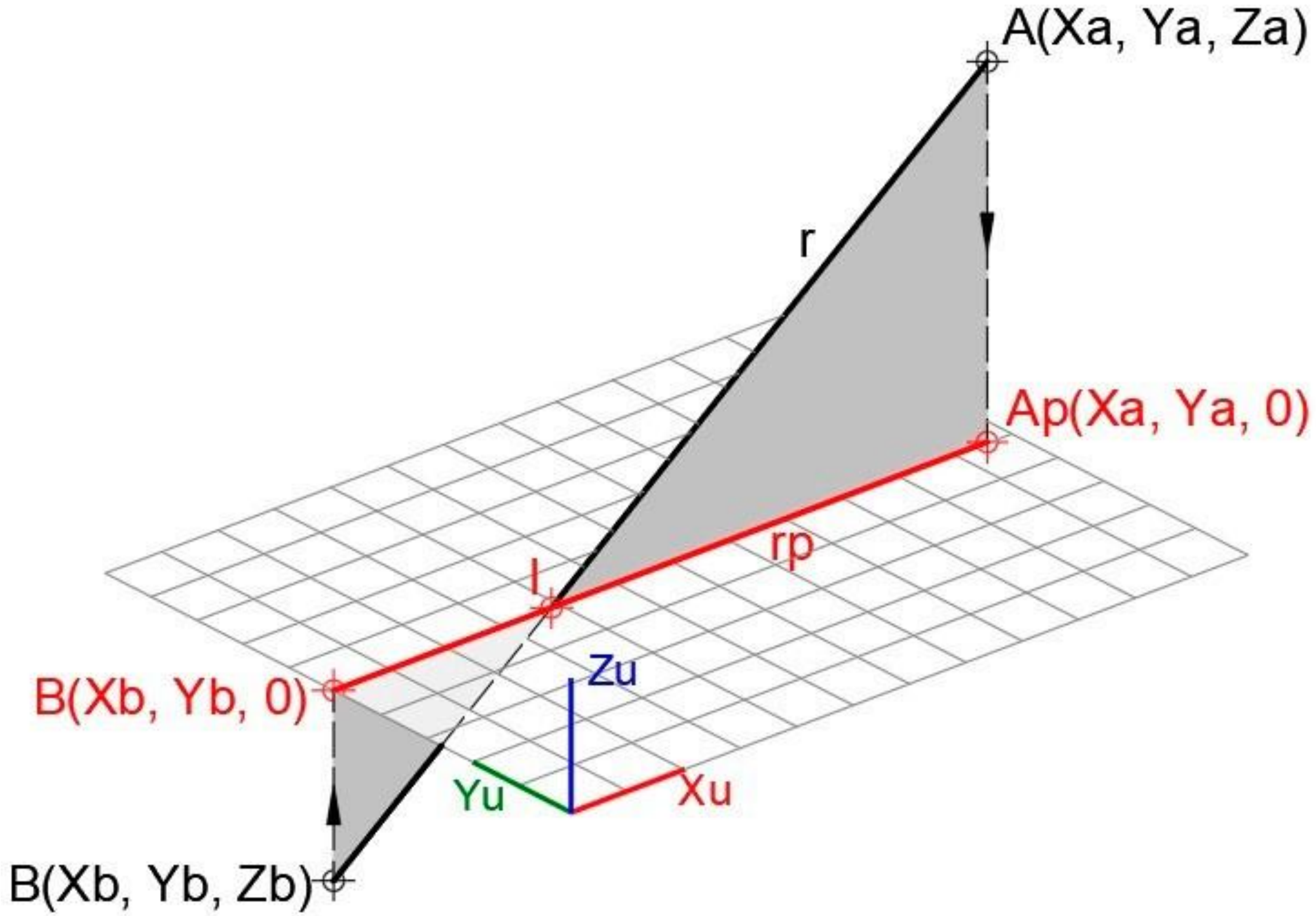

Figure 20. Checking whether line *r* and meshed plane are parallel.

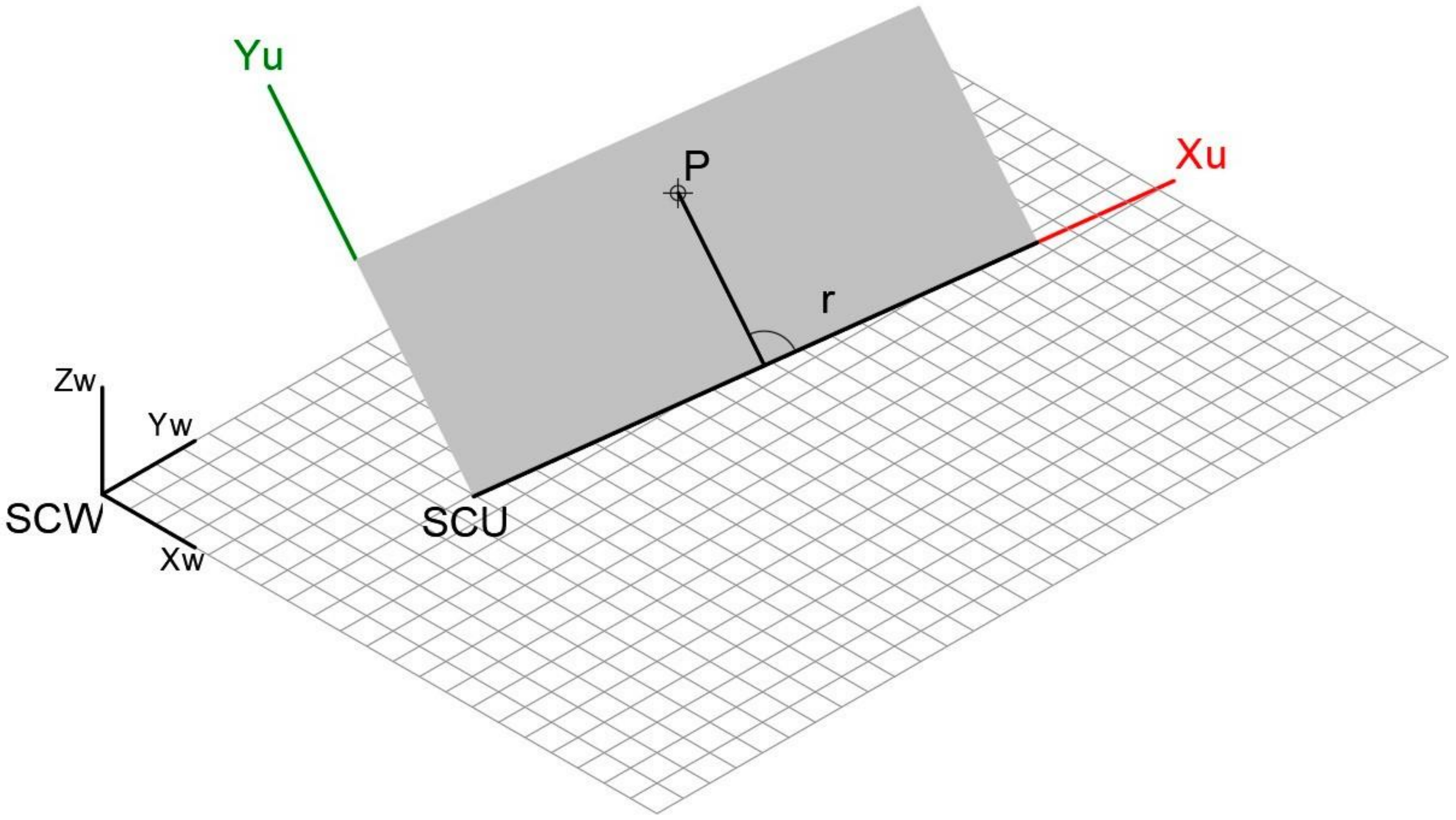

Figure 21. Perpendicular line from point P to line *r* by using UCS.

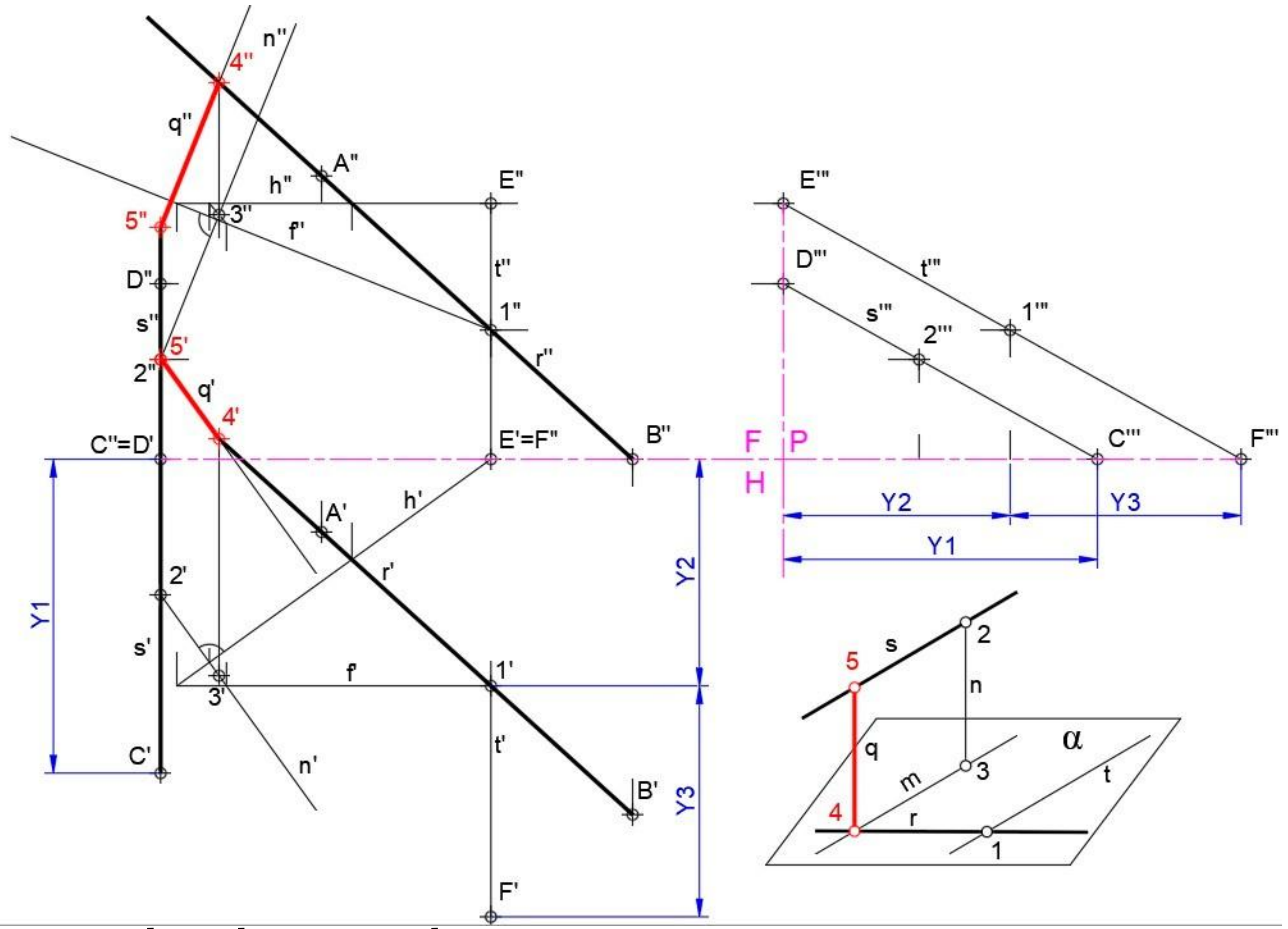

Figure 22. Determining the perpendicular line and shortest distance between two skew lines, *r* and *s*, applying the traditional method. A sketch of the followed procedure is shown on the bottom right corner.

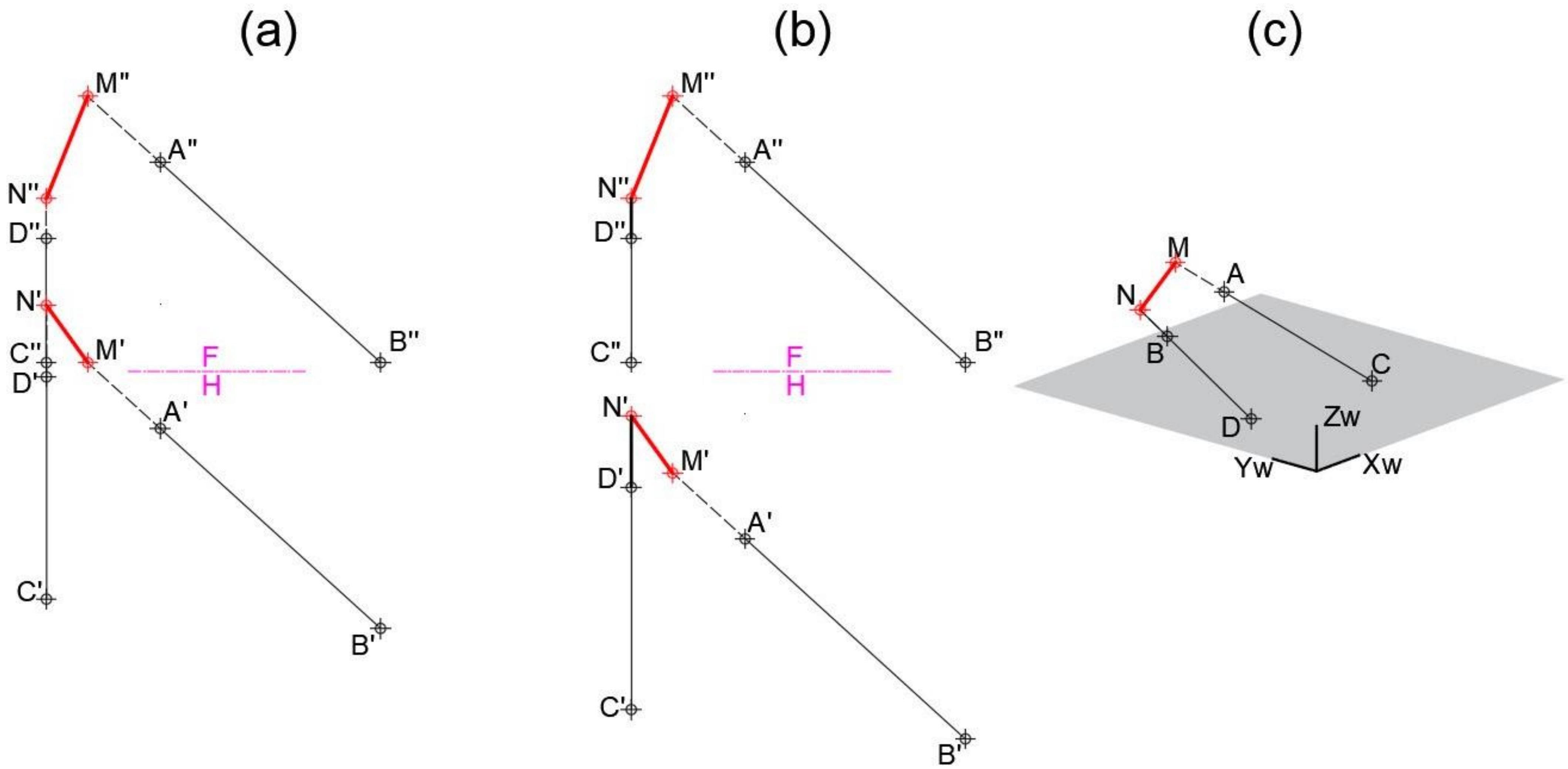

Figure 23. Solution obtained using CADOP for the perpendicular line and shortest distance between two skew lines *r* and *s*. (**a**) Hiding the layers in which intermediate steps are drawn. (**b**) Arranging the views for avoiding projections overlapping. (**c**) Three-dimensional view.

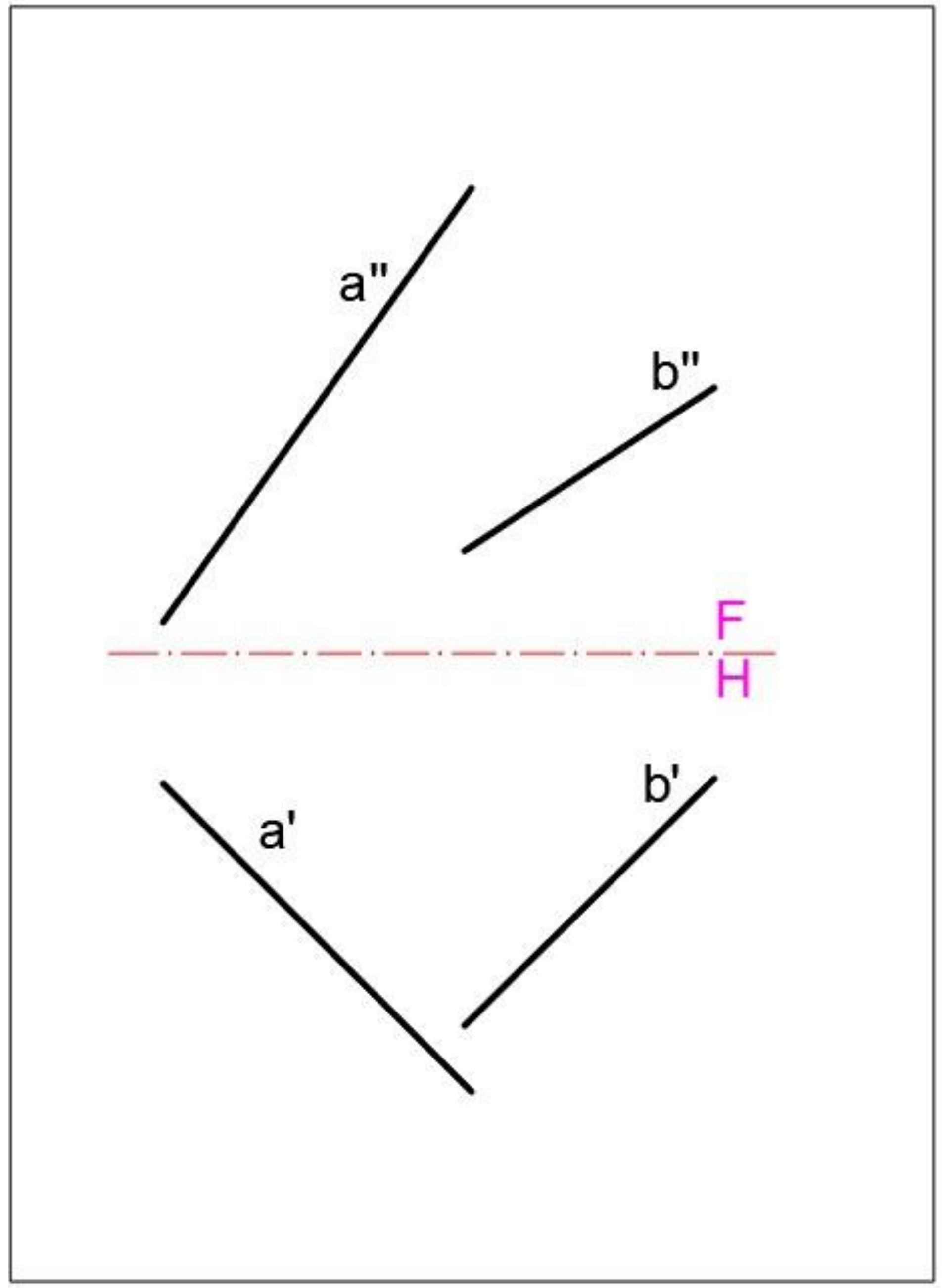

Figure 24. Problem statement: the top and front views of skew lines *a* and *b*.

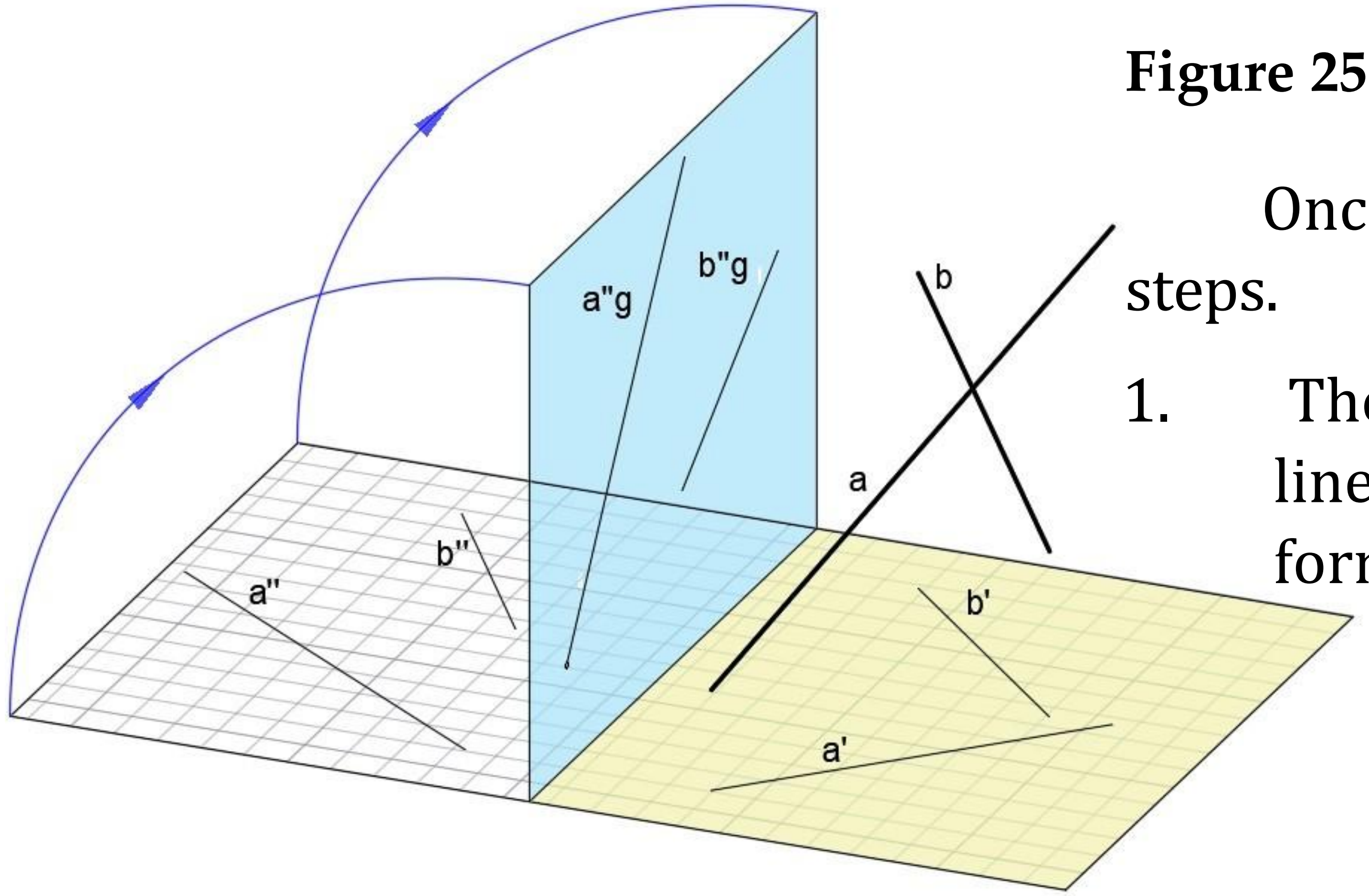

Figure 25. Line *a* and *b* restitution for obtaining their 3D positions.

Once the lines are restored, the resolution of the problem is described in the following steps.

1. The plane is defined through line *b* parallel to line *a* (Figure 26). Line *a*1 is a copy of line *a* through any point *P* on line b. The plane is determined by obtaining the UCS formed by line *a*1 and *b*.

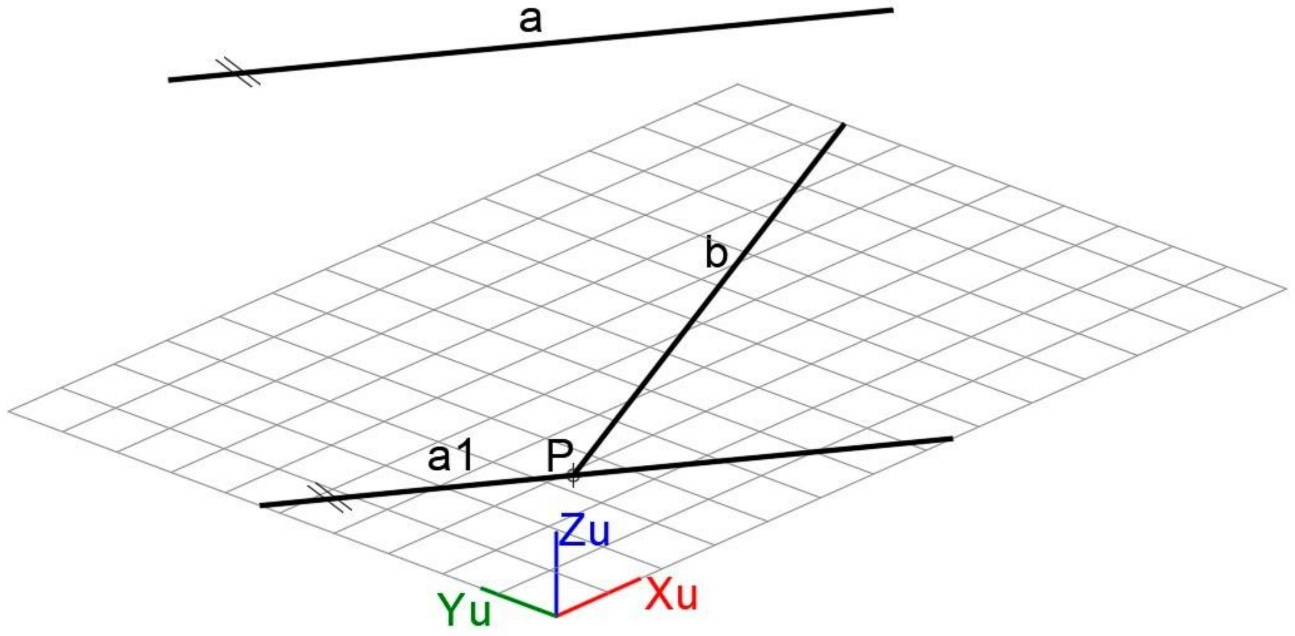

Figure 26. Plane through line *b* parallel to line *a*.

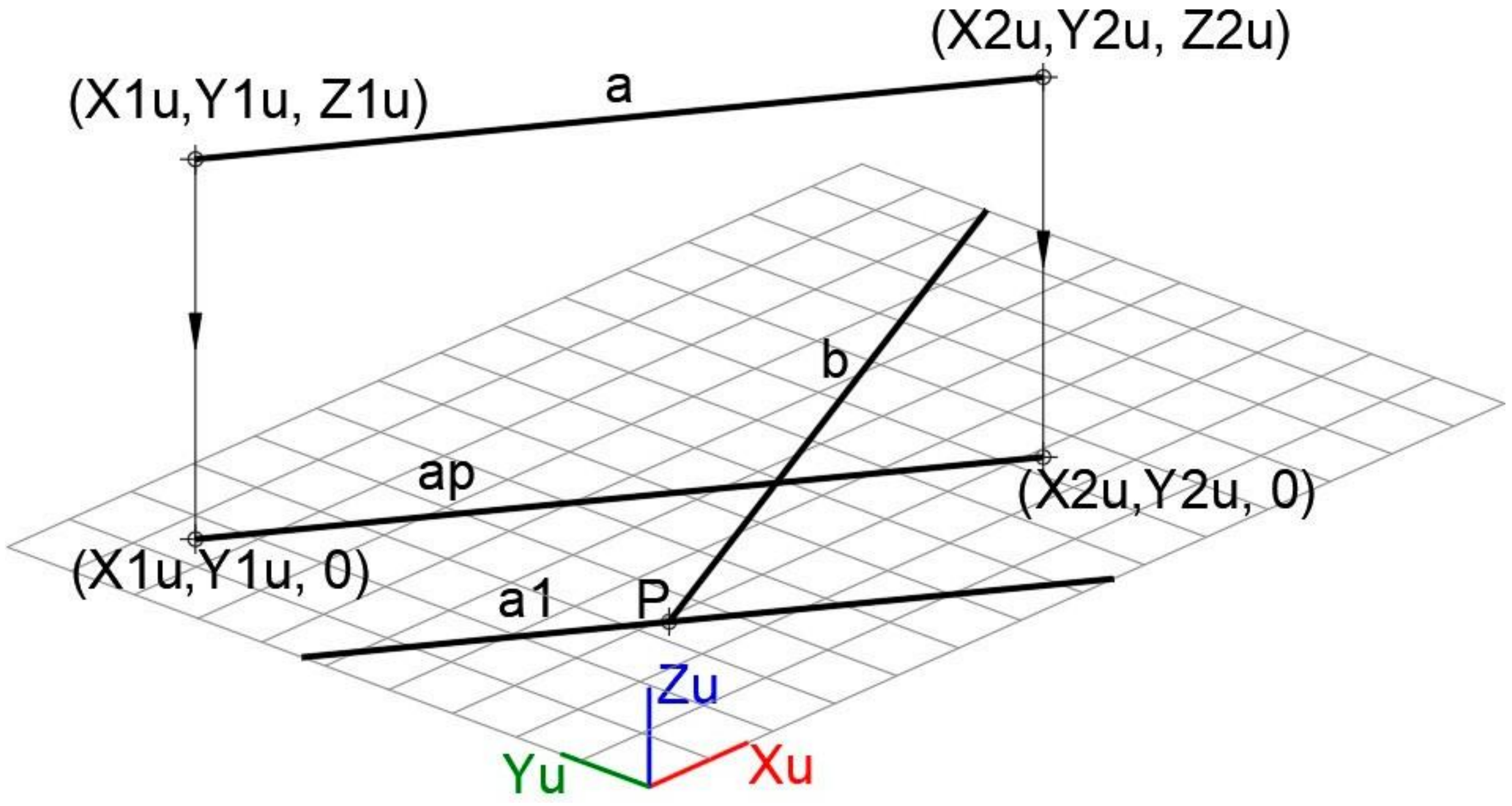

Figure 27. Projection *ap* of line *a* onto the plane defined by *a*1 and *b*.

2. Making Zu = 0 for any two points on line *a* results in a projection onto the UCS defined before. Figure 27 shows line *ap* resulting from the projection.

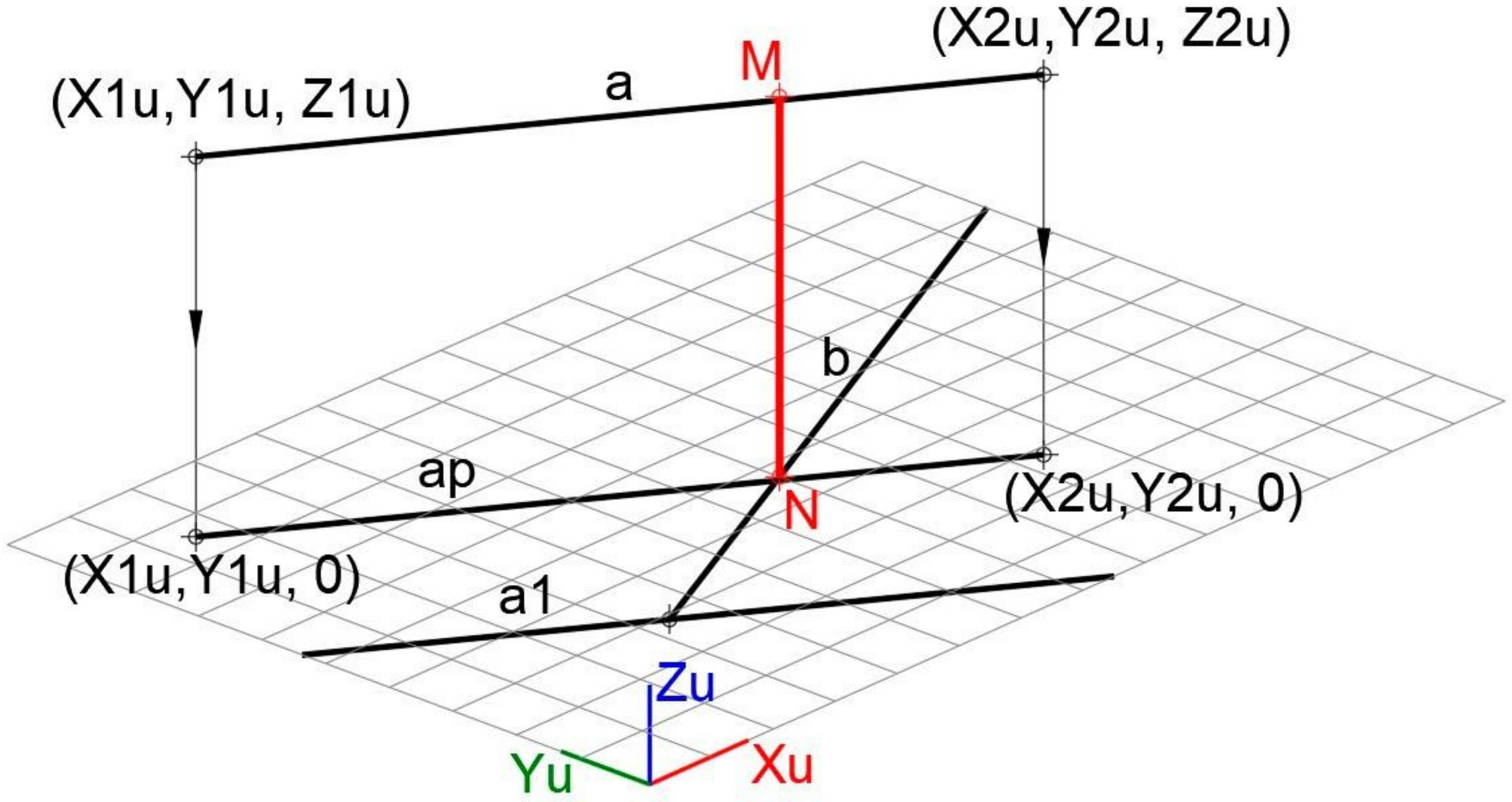

Figure 28. Perpendicular line and shortest distance segment *MN* between skew lines *a* and *b*.

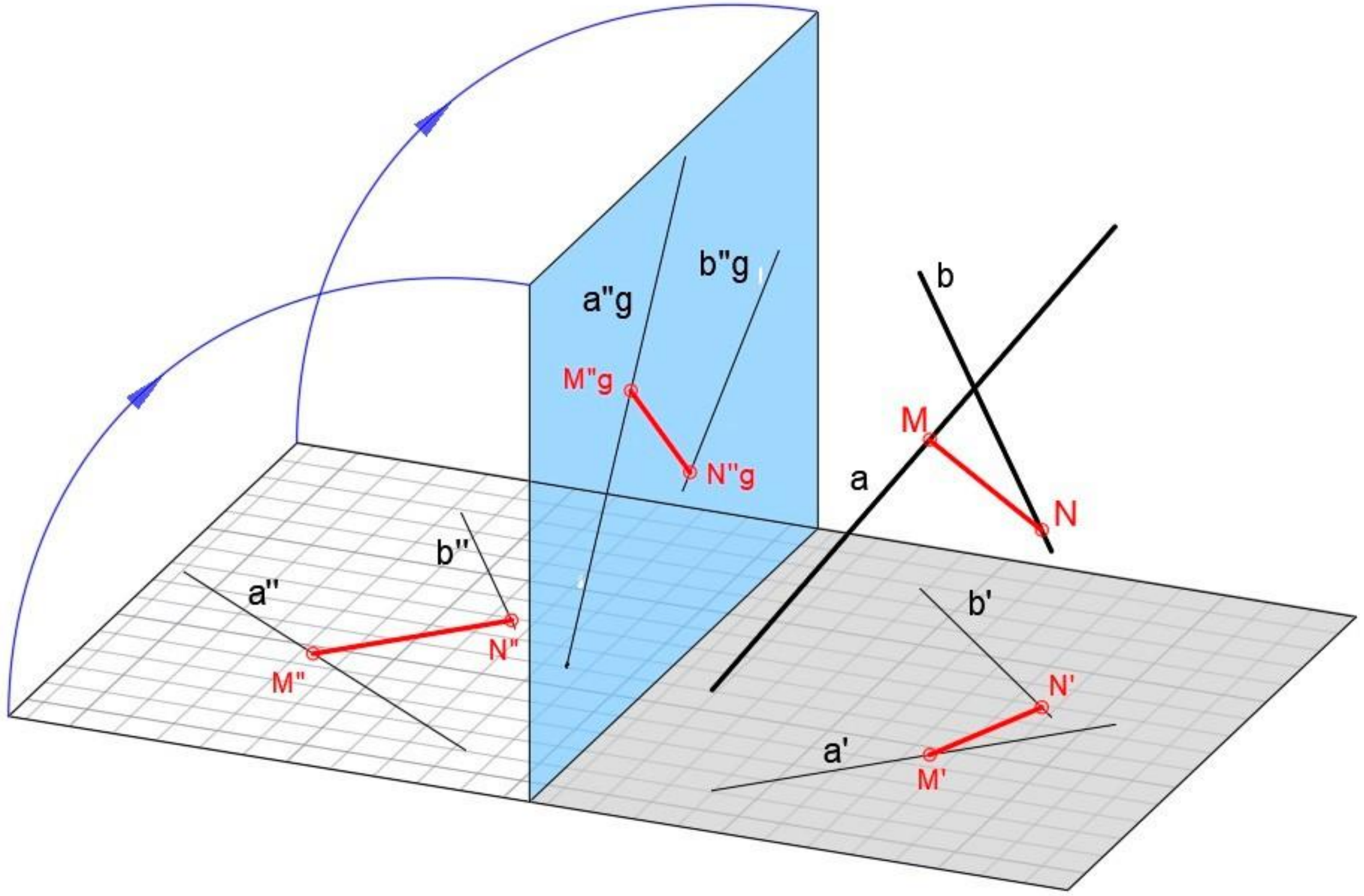

Figure 29. Three-dimensional position of perpendicular line and shortest distance segment *MN* between skew lines *a* and *b*.

The *MN* segment's 3D position obtained with CADOP as a solution to the proposed problem is shown in Figure 29.

4. Finally, the top and front views and true length dimension of segment *MN* are ob- tained. Figure 30 shows the solution provided by applying CADOP.

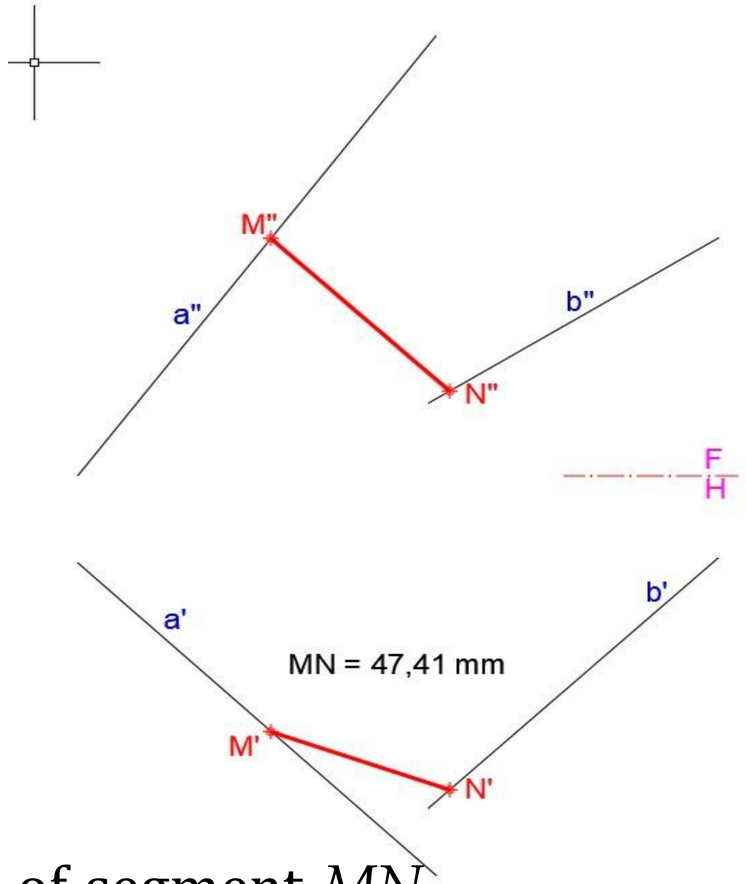

Figure 30. Top and front views and true length dimension of segment *MN*.

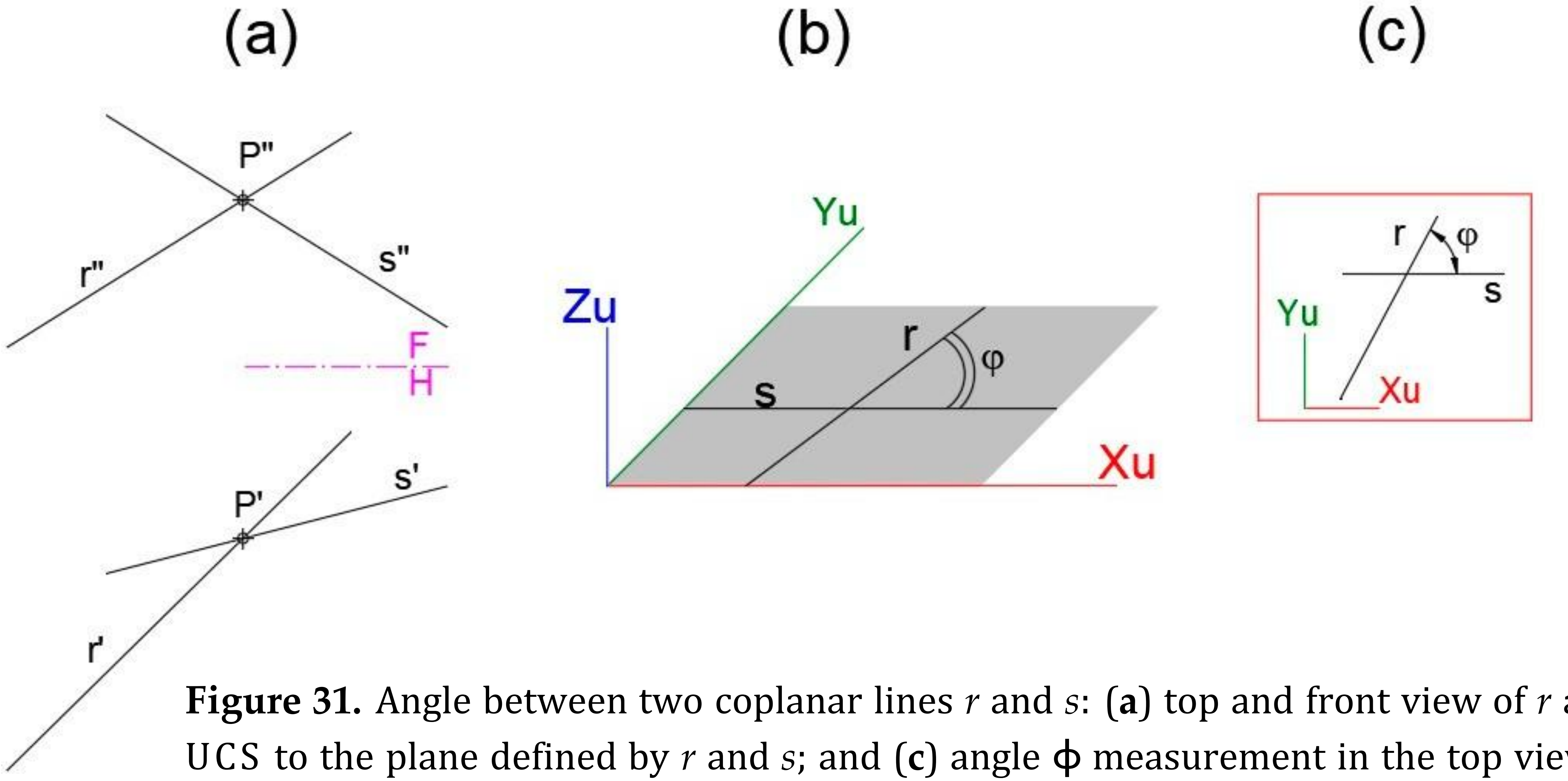

Figure 31. Angle between two coplanar lines *r* and *s*: (**a**) top and front view of *r* and *s*; (**b**) matching UCS to the plane defined by *r* and *s*; and (**c**) angle ϕ measurement in the top view of the UCS.

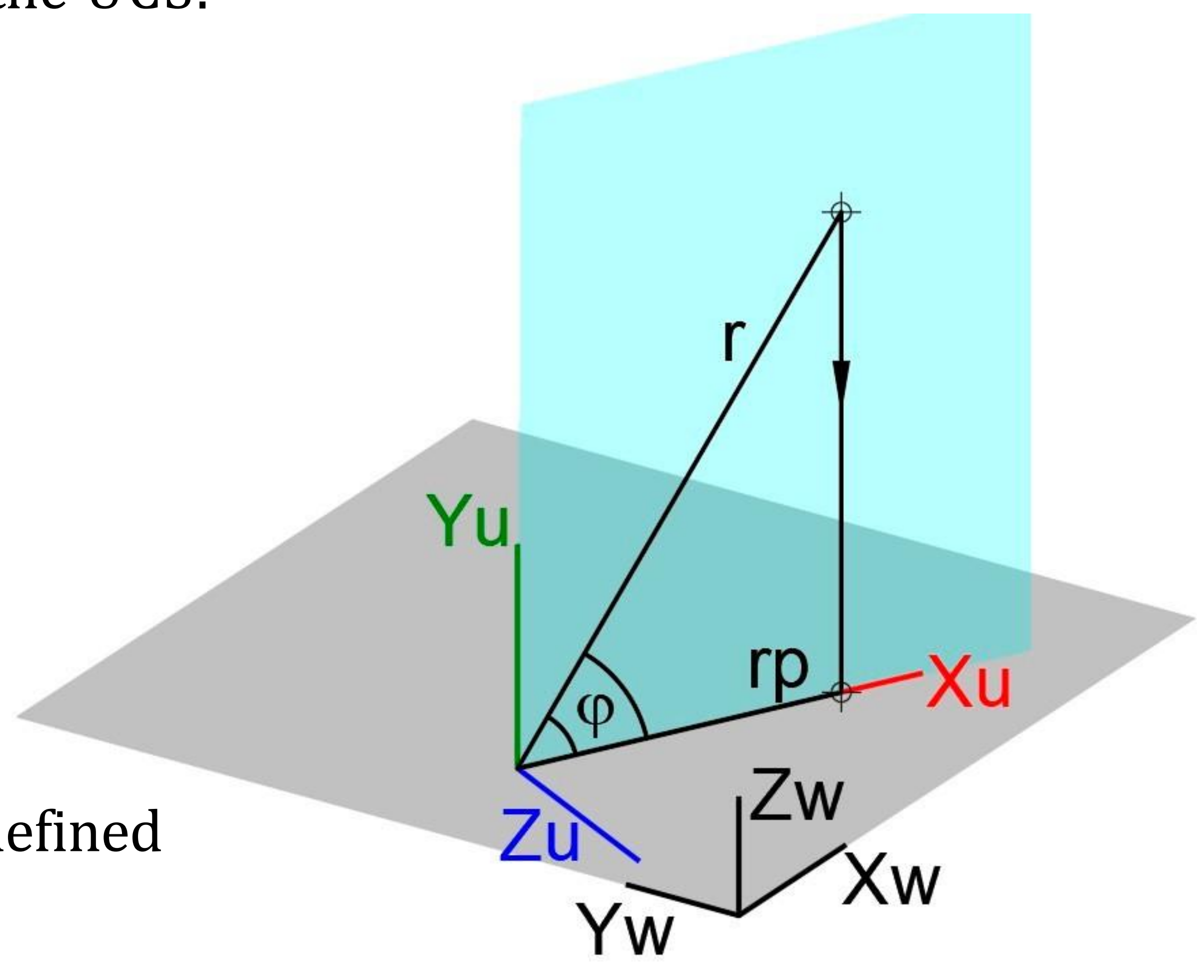

Figure 32. Angle between line and plane: angle ϕ measurement in the top view of the UCS defined by line *r* and its projection *rp* onto a plane.

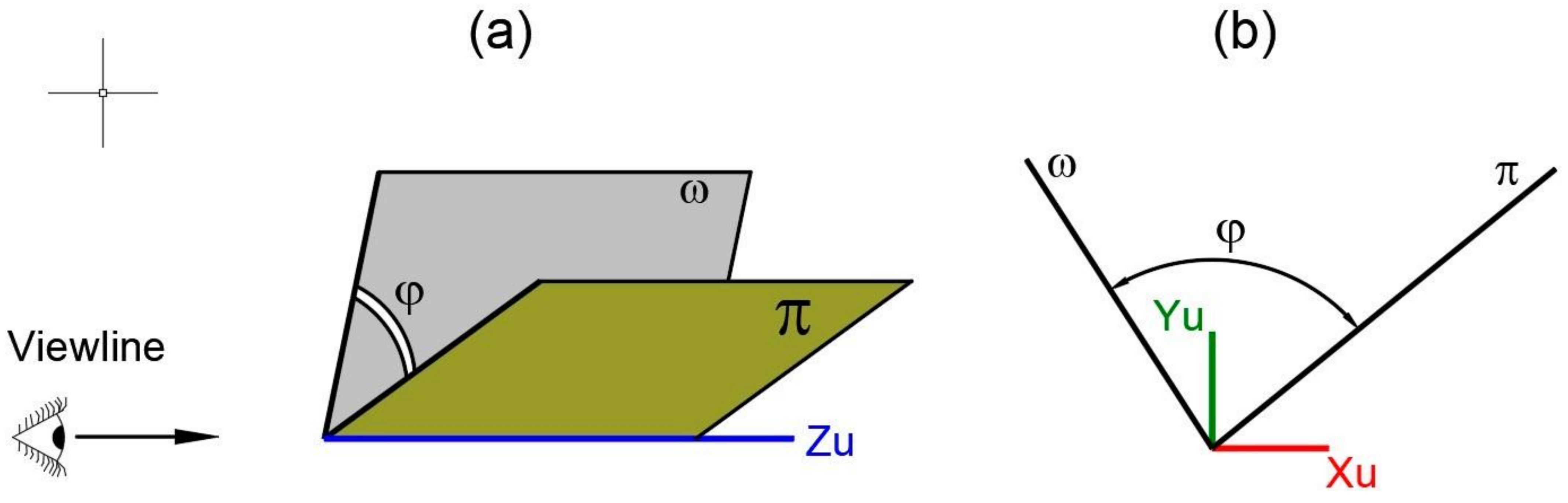

Figure 33. Angle ϕ between two planes π and ω: (**a**) UCS Z axis matches to the intersection line of both planes; (**b**) angle ϕ measurement is shown in top view.

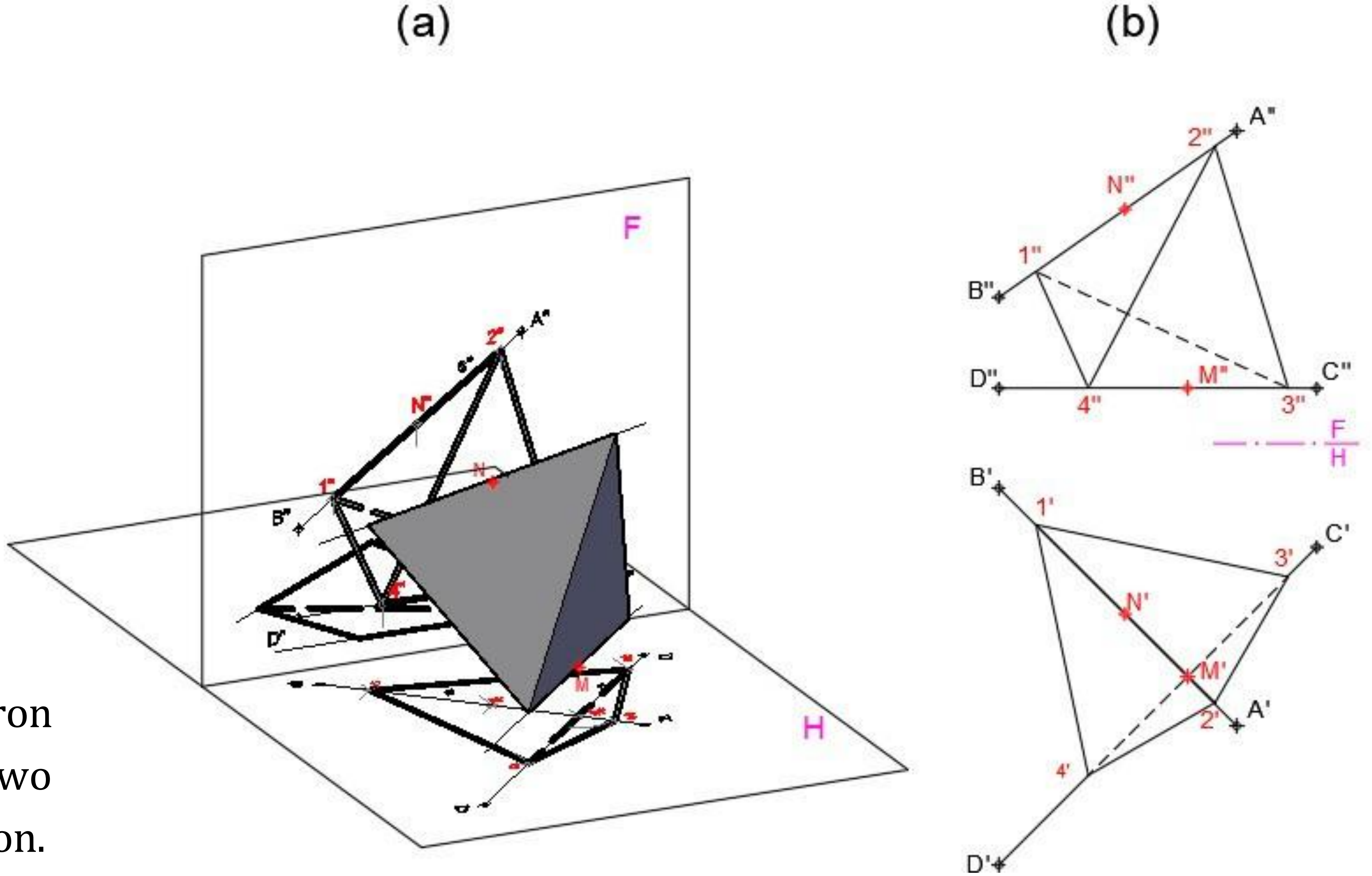

Figure 34. Regular polyhedra problem solved with CADOP: (**a**) tetrahedron obtained in 3D, knowing that two of its opposite edges are located on two perpendicular skew lines; (**b**) top and front views of the polyhedron solution.