جامعة العلوم والتقانة
University of Science and Technology

Faculty of Engineering

Calculus and it's Applications -1

Lecture - 7

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$$y = cos(u) \quad \Rightarrow \quad y' = -u' sin(u)$$

$$y = sin(u) \quad \Rightarrow \quad y' = u' cos(u)$$

$$y = tan(u) \quad \Rightarrow \quad y' = u' sec^2(u)$$

$$y = cot(u) \quad \Rightarrow \quad y' = -u' csc^2(u)$$

$$y = sec(u) \quad \Rightarrow \quad y' = u' sec(u)\tan(u)$$

$$y = csc(u) \quad \Rightarrow \quad y' = -u' csc(u)\cot(u)$$

$$y = ln(u) \quad \Rightarrow \quad y' = \frac{u'}{u}$$

$$y = ln(x^2) \rightarrow \quad y' = \frac{2x}{x^2}$$

$$y = \log_a(u) \quad \Rightarrow \quad y' = \frac{u'}{lna \;\; u}$$

$$y = \log_3 sinx \implies y' = \frac{cosx}{ln3\ sinx}$$

$$y = e^u \quad \Rightarrow \quad y' = u'(e^u)$$

$$y = e^{tanx} \rightarrow y' = sec^2 x e^{tanx}$$

$$y = a^u \quad \Rightarrow \quad y' = u' lna(a^u)$$

Excersise :

Show that if:

$$y = tan(u) \quad \Rightarrow \quad y' = u' sec^2(u)$$

$$y = cot(u) \quad \Rightarrow \quad y' = -u' csc^2(u)$$

$$y = sec(u) \quad \Rightarrow \quad y' = u' sec(u)\tan(u)$$

$y = csc(u) \quad \Rightarrow \quad y' = -u' csc(u)\cot(u)$

$y = e^u \quad \Rightarrow \quad y' = u'(e^u)$

$y = a^u \quad \Rightarrow \quad y' = -u' \, lna(\, a^u)$

Examples : Find $\frac{dy}{dx}$ of the following functions

$(i)\ y = cos(x^2 + x) \quad y' = -(2x + 1)\sin(x^2 + x)$

$(ii)\ y = sin(5x) \quad \Rightarrow \quad y' = 5\cos(5x)$

$(iii)\ y = tan(\sin(x^2)) \quad \Rightarrow \quad y' = 2xcos(x^2)\, sec^2(sin(x^2))$

$(iv)\ y = cot(2 + 2x) \quad \Rightarrow \quad y' = -2\, csc^2(2 + 2x)$

$(iv)\ y = sec(\sqrt{x}) \quad \Rightarrow \quad y' = \frac{1}{2}\frac{1}{\sqrt{x}} \sec(\sqrt{x})tan(\sqrt{x})$

$(v)\ y = csc(\text{tanx}) \quad \Rightarrow \quad y' = -\sec^2(x)csc(tanx)cot(tanx)$

$(vi)\ y = ln(x^7 + 1) \quad \Rightarrow \quad y' = \frac{7x^6}{x^7 + 1}$

$(vii)\ y = \log_5(x^7 + 1) \quad \Rightarrow \quad y' = \frac{7x^6}{(x^7 + 1)ln5}$

$(viii)\ y = e^{xsinx} \quad \Rightarrow \quad y' = (xcosx + sinx\ )\, e^{xsinx}$

$(viiii)\ y = 7^{x^2} \quad \Rightarrow \quad y' = 2x\, ln7\, e^{xsinx}$

Examples : Show that following functions

$(i)\ y = tanx \quad \Rightarrow \quad y' = sec^2 x$

Proof

$$y = tanx = \frac{sinx}{cosx} = \frac{cosx\ cosx + sinx\ sinx}{cos^2x} = \frac{cos^2x + sin^2x}{cos^2x}$$
$$= \frac{1}{cos^2x} = sec^2x$$

$(ii)\ y = e^u \quad \Rightarrow \quad y^{\backslash} = u^{\backslash} e^u$

Proof

$$y = e^u \Rightarrow lny = lne^u = u\ lne = \Rightarrow = \frac{1}{y} y^{\backslash} = u^{\backslash} lne \Rightarrow y^{\backslash} = u^{\backslash} y$$
$$= u^{\backslash} e^u$$

$(iii)\ y = a^u \quad \Rightarrow \quad y^{\backslash} = u^{\backslash} lna\ a^u$

Proof

$$y = a^u \Rightarrow lny = lna^u = u\ lna = \Rightarrow = \frac{1}{y} y^{\backslash} = u^{\backslash} lna \Rightarrow y^{\backslash} = u^{\backslash}\ lna\ y$$
$$= u^{\backslash}\ lna\ a^u$$

$(iv)\ y = sinhx \quad \Rightarrow \quad y^{\backslash} = coshx$

Proof

$$y = sinhx = \frac{e^x - e^{-x}}{2} = \frac{1}{2}(e^x - e^{-x}) \quad \Rightarrow \quad y^{\backslash} = \frac{1}{2}(e^x + e^{-x})$$
$$= \frac{e^x + e^{-x}}{2} = coshx$$

$(v)\ y = cos^{-1}x \quad \Rightarrow \quad y^{\backslash} = \frac{-1}{\sqrt{1 - x^2}}$

Proof

$$y = cos^{-1}x = x = cosy \quad \Rightarrow \quad 1 = -siny\, y^{\backslash} = y^{\backslash} = \frac{-1}{siny}$$

Take $siny = \sqrt{sin^2 y}$

$$y^{\backslash} = \frac{-1}{siny} = \frac{-1}{\sqrt{sin^2 y}} = \frac{-1}{\sqrt{1 - cos^2 y}} = \frac{-1}{\sqrt{1 - x^2}}$$

Excersise :

Find $y^{\backslash}$ of the following functions

$(i)\ y = sin^{-1}\left(\frac{x}{cosx}\right)$ $(ii)\ y = coth(lnx)$ $(iii)\ y = cos^9 x$

$(iv)\ y = ln\left(\frac{x^2}{x+3}\right)$ $(v)\ y = x^9 sec\left(\frac{1}{x}\right)$ $(vi)\ y = sin3x\, cos2x$

$(vii)\ y = e^{\sin^{-1}2x}$ $(viii)\ y = 9^{9x^2+2}$ $(viiii)\ y = \log_7 \frac{1}{\sqrt[3]{x}}$

2 - show that following functions

$(i)\ y = \sec^{-1} x \Rightarrow y^{\backslash} = \frac{1}{x\sqrt{x^2-1}}$ $(ii)\ y = \csc^{-1} x \Rightarrow y^{\backslash} = \frac{-1}{x\sqrt{x^2-1}}$

$(iii)\ y = \cot^{-1} x \Rightarrow y^{\backslash} = \frac{-1}{1+x^2}$ $(iv)\ y = coshx \Rightarrow y^{\backslash} = sinhx$

$$(iv)\ y^2 = xcosy \quad \Rightarrow \quad 2y^{\backslash} = -xy^{\backslash}siny + cosy \Rightarrow y^{\backslash}(2 - xsiny) = cosy \Rightarrow y^{\backslash} = \frac{cosy}{(2 - xsiny)}$$

$$(v)\ y = cos^{-1}x \quad \Rightarrow \quad x = cosy \Rightarrow 1 = y^{\backslash}cosy \Rightarrow y^{\backslash} = \frac{1}{cosy}$$

$$y^{\backslash} = \frac{1}{\sqrt{cos^2 y}} = \frac{1}{\sqrt{1 - sin^2 y}} = \frac{1}{\sqrt{1 - x^2}}$$

Where

$$sin^2 y + cos^2 y = 1$$