جامعة العلوم والتقانة

University of Science and Technology

Faculty of Engineering

Calculus and it’s Applications -1

Lecture - Five

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Rules of Differentiation

$$\frac{dy}{dx} \cdot \; y^{\backslash}, \;\; f^{\backslash}(x)$$

(1) $\frac{dk}{dx} = 0$, k ≡ constant (2) $\frac{dx}{dx} = 1$

(3) $\frac{d(kx)}{dx} = k$ (4) $\frac{dx^n}{dx} = nx^{n-1}$

(5) $\frac{d\left(kf(x)\right)}{dx} = k\frac{df(x)}{dx} = kf^{\backslash}(x)$

(6) $\frac{d(\,f(x) \pm g(x))}{dx} = \frac{df(x)}{dx} \pm \frac{dg(x)}{dx} = f^{\backslash}(x) \pm g^{\backslash}(x)$

(7) $\frac{d(\,f(x).g(x))}{dx} = f(x)\frac{dg(x)}{dx} + g(x)\frac{df(x)}{dx} = f(x).g^{\backslash}(x) \pm g(x)f^{\backslash}(x)$

(8) $\frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{(g(x))^2} = \frac{g(x)f^{\backslash}(x) - f(x)g^{\backslash}(x)}{(g(x))^2}$

(9) $\frac{d\left(f(x)\right)^n}{dx} = n(f(x))^{n-1}\, f^{\backslash}(x)$

Example:

Find $\frac{dy}{dx}$ of the following functions

(1) $y = f(x) = x^2 + 9x - 9$

Solution:

$$y^{\backslash} = 2x + 9$$

(2) $y = f(x) = (x^2 - 4x)^7$

Solution:

$$y^{\backslash} = 7(x^2 - 4x)^6(2x - 4)$$

(3) $y = f(x) = x^2(x^5 - 8)$

Solution:

$$y^{\backslash} = x^2(5x^4) + 2x(x^5 - 8)$$

(4) $y = x^2 sinx$

Solution:

$$y^\backslash = x^2 cosx + 2x\ sinx$$

(5) $y = \frac{cosx}{x^2}$

Solution:

$$y^\backslash = \frac{x^2(-sinx) - 2x\ cosx}{x^4} = \frac{-x^2 sinx - 2x\ cosx}{x^4}$$

(5) $y = tanx = \frac{sinx}{cosx}$

Solution:

$$y^\backslash = \frac{cosx\ \ cosx + sinx\ sinx}{cos^2x} = \frac{cos^2x + sin^2x}{cos^2x} = \frac{1}{cos^2x} = sec^2x$$

$$y = cos(u) \quad \Rightarrow \quad y^\backslash = -u^\backslash sin(u)$$

$$y = sin(u) \quad \Rightarrow \quad y^\backslash = u^\backslash cos(u)$$

$$y = tan(u) \quad \Rightarrow \quad y^\backslash = u^\backslash sec^2(u)$$

$$y = cot(u) \quad \Rightarrow \quad y^\backslash = -u^\backslash csc^2(u)$$

$$y = sec(u) \quad \Rightarrow \quad y^\backslash = u^\backslash sec(u)\tan(u)$$

$$y = csc(u) \quad \Rightarrow \quad y^\backslash = -u^\backslash csc(u)\cot(u)$$

$$y = ln(u) \quad \Rightarrow \quad y^\backslash = \frac{u^\backslash}{u}$$

$$y = \log_a(u) \quad \Rightarrow \quad y^\backslash = \frac{u^\backslash}{lna\ \ u}$$

$$y = e^{u} \quad \Rightarrow \quad y^{\backslash} = u^{\backslash}(e^{u})$$

$$y = a^{u} \quad \Rightarrow \quad y^{\backslash} = u^{\backslash}\, lna(\, a^{u})$$

Excersise :

Show that if:

$y = tan(u) \quad \Rightarrow \quad y^{\backslash} = u^{\backslash} sec^{2}(u)$

$y = cot(u) \quad \Rightarrow \quad y^{\backslash} = -u^{\backslash} csc^{2}(u)$

$y = sec(u) \quad \Rightarrow \quad y^{\backslash} = u^{\backslash} sec(u)\tan(u)$

$y = csc(u) \quad \Rightarrow \quad y^{\backslash} = -u^{\backslash} csc(u)\cot(u)$

$y = e^{u} \quad \Rightarrow \quad y^{\backslash} = u^{\backslash}(e^{u})$

$y = a^{u} \quad \Rightarrow \quad y^{\backslash} = -u^{\backslash}\, lna(\, a^{u})$

Exercise

1. Find the $\frac{dy}{dx}$ of the following functions

(1) $f(x) = x^{2} - 3x + 9$ (2) $f(x) = cotx - tanx$

(3) $f(x) = \sqrt{4 - 2x^{2}}$ (4) $f(x) = \ln(6 - 5x + x^{2})$

(5) $f(x) = \sqrt{|2x - 5|}$ (6) $f(x) = \frac{2x}{x^{2-2x+1}}$

2. Which of the following functions are find $\frac{dy}{dx}$?

(1) $f(x) = x^{6} + 1$ (2) $f(x) = xcotx$ (3) $f(x) = x^{2} \sec x$

(4) $f(x) = \frac{tanx}{x^{2}}$ (5) $f(x) = x^{2} - x +$ (6) $f(x) = sinhx$

Exercise:

Find $\frac{dy}{dx}$

(1) $y = f(x) = \sqrt{x}$ (2) $y = f(x) = x^2 - 3x + 4$

(3) $y = f(x) = \frac{1}{x-5}$ (4) $y = f(x) = cosx$

(5) $y = f(x) = \sqrt[3]{x+3}$ (6) $y = f(x) = e^x$ (7) $y = f(x) = \log_3 x^2$