جامعة العلوم والتقانة
University of Science and Technology

Faculty of Engineering

Calculus and it's Applications -1

Lecture - 10

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Examples:

Prove the following equations if:

1) $y = \frac{1}{1+x}$ $\quad$ *prove that* $\quad y^{\backslash\backslash\backslash} + 6y^4 = 0 \ldots\ldots\ldots\ldots(1)$

Solution

$$y = \frac{1}{1+x} = (1+x)^{-1} = \qquad y^{\backslash} = -(1+x)^{-2} \qquad y^{\backslash\backslash} = 2(1+x)^{-3}$$

$$y^{\backslash\backslash\backslash} = -6(1+x)^{-4} = \frac{-6}{(1+x)^4}$$

$$\frac{-6}{(1+x)^4} + 6\left(\frac{1}{1+x}\right)^4 = \frac{-6}{(1+x)^4} + \frac{6}{(1+x)^4} = 0$$

2) $y = \frac{sinhx}{coshx - sinhx}$ $\quad$ *prove that* $\quad y^{\backslash} = e^{2x}$

Solution

$$y^{\backslash} = \frac{(coshx - sinhx)coshx - \sinh x(sinhx - coshx)}{(coshx - sinhx)^2} =$$

$$y^{\backslash} = \frac{cosh^2x - \sinh x \cosh x - sinh^2x + sinhx\ coshx}{(coshx - sinhx)^2} = \frac{cosh^2x - sinh^2x}{(coshx - sinhx)^2} = \frac{1}{(coshx - sinhx)^2}$$

Where $cosh^2x - sinh^2x = 1$

$$coshx - sinhx = \frac{e^x + e^{-x}}{2} - \left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x + e^{-x} - e^x + e^{-x}}{2} = \frac{2e^{-x}}{2} = e^{-x}$$

$$y^{\backslash} = \frac{1}{(coshx - sinhx)^2} = \frac{1}{(e^{-x})^2} = e^{2x}$$

Example (1):

Find:

(1) $\lim\limits_{x\to 0} x^2 + 4x - 5$ $\qquad$ (2) $\lim\limits_{x\to 1} \sqrt{2x+4}$ $\qquad$ (3) $\lim\limits_{x\to 0} \cos x + x^2$

(4) $\lim\limits_{x\to 0} \ln(x+7)$ $\qquad$ (5) $\lim\limits_{x\to 3} \frac{x^2+1}{x-2}$ $\qquad$ (6) $\lim\limits_{x\to 0} 9$

Solution

(1) $\lim\limits_{x\to 0} x^2+4x-5 = 0^2+4(0)-5 = -5$

(2) $\lim\limits_{x\to 1}\sqrt{2x+4} = \sqrt{2+4} = \sqrt{6}$

(3) $\lim\limits_{x\to 0} \cos x + x^2 = cos0 - 0^2 = 1-0 = 1$

(4) $\lim\limits_{x\to 0} \ln(x+7) = \ln(0+7) = ln7$

(5) $\lim\limits_{x\to 3}\frac{x^2+1}{x-2} = \frac{9+1}{3-2} = 10$

(6) $\lim\limits_{x\to 0} 9 = 9$

Example (2):

If $f(x) = \begin{cases} 2x-1\,, & x \geq 1 \\ x^2\,, & x < 2 \end{cases}$ Find:

(1) $\lim\limits_{x\to 1^+} \mathrm{f}(x)$ (2) $\lim\limits_{x\to 1} \mathrm{f}(x)$ (3) $\lim\limits_{x\to 1} \mathrm{f}(x)$

Solution

(1) $\lim\limits_{x\to 1^+} \mathrm{f}(x) = \lim\limits_{x\to 1} 2x-1 = 2-1 = 1$

(2) $\lim\limits_{x\to \bar{1}} \mathrm{f}(x) = \lim\limits_{x\to \bar{1}} \mathrm{x}^2 = 1^2 = 1$

(3) $\lim\limits_{x\to 1} \mathrm{f}(x) = 1 \qquad \left(\lim\limits_{x\to 1^+} \mathrm{f}(x) = \lim\limits_{x\to \bar{1}} \mathrm{f}(x)\right) = 1$

Exercise – 3

1. Find the domain of definition of the following functions

(1) $f(x) = x^2-3x+9$ (2) $f(x) = cotx - tanx$

(3) $f(x) = \sqrt{4-2x^2}$ (4) $f(x) = \ln(6-5x+x^2)$

(5) $f(x) = \sqrt{|2x-5|}$ (6) $f(x) = \frac{2x}{x^{2-2x+1}}$

2. Which of the following functions are even and which are odd?

(1) $f(x) = x^6+1$ (2) $f(x) = xcotx$ (3) $f(x) = x^2\sec x$

(4) $f(x) = \frac{tanx}{x^2}$ (5) $f(x) = x^2 - x +$ (6) $f(x) = sinhx$

Indeterminate Forms

Indeterminate forms

theorems or mathematical operations, for example factoring, multiplying by the conjugate, dividing numerator and denominator by the highest power of x.

Note:

$\equiv$ any number

Example: