

University of Science and Technology

Engineering of electronic



Physics II

COURSE COORDINATOR

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Lecture seven

Alternating Current

Alternating Current

THE EMF GENERATED BY A ROTATING COIL in a magnetic field has a graph similar to the one shown in Fig. 35-1. It is called an *ac voltage* because there is a reversal of polarity (i.e., the voltage changes sign); ac voltages need not be sinusoidal. If the coil rotates with a frequency of f revolutions per second, then the emf has a frequency of f in hertz (cycles per second). The instantaneous voltage v that is generated has the form

$$v = v_0 \sin \omega t = v_0 \sin 2\pi f t$$

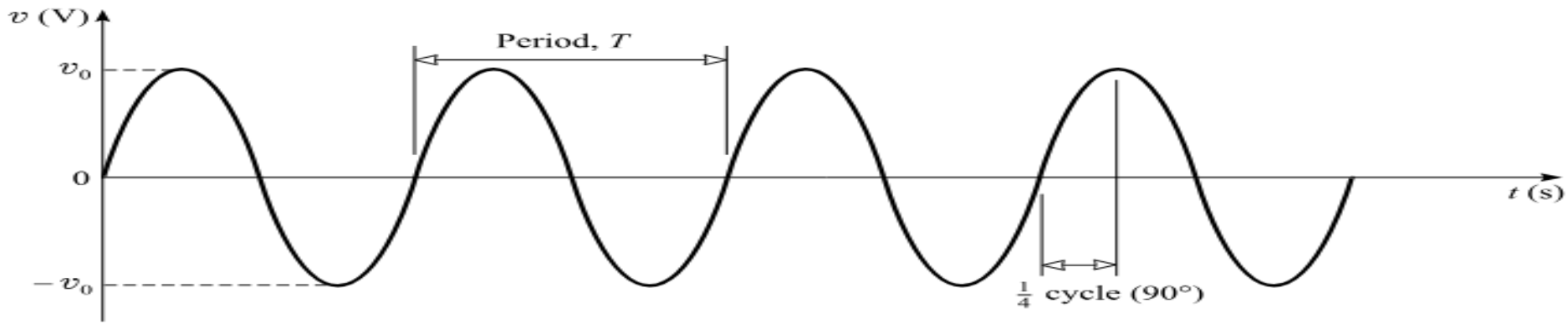


Fig. 35-1

where v_0 is the amplitude (maximum value) of the voltage in volts, $\omega = 2\pi f$ is the angular velocity in rad/s, and f is the frequency in hertz. The frequency f of the voltage is related to its period T by

$$T = \frac{1}{f}$$

where T is in seconds.

Rotating coils are not the only source of ac voltages; electronic devices for generating ac voltages are very common. Alternating voltages produce alternating currents.

An alternating current produced by a typical generator has a graph much like that for the voltage shown in Fig. 35-1. Its instantaneous value is i, and its amplitude is i_0. Often the current and voltage do not reach a maximum at the same time, even though they both have the same frequency.

METERS for use in ac circuits read the *effective*, or *root mean square* (rms), values of the current and voltage. These values are always positive and are related to the amplitudes of the instantaneous sinusoidal values through

$$V = V_{\text{rms}} = \frac{v_0}{\sqrt{2}} = 0.707 v_0$$

$$I = I_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$

It is customary to represent meter readings by capital letters (V, I), while instantaneous values are represented by small letters $(v,\ i)$.

THE THERMAL ENERGY GENERATED OR POWER LOST by an rms current I in a resistor R is given by I^2R.

FORMS OF OHM'S LAW: Suppose that a sinusoidal current of frequency f with rms value I flows through a pure resistor R, or a pure inductor L, or a pure capacitor C. Then an ac voltmeter placed across the element in question will read an rms voltage V as follows:

$$\text{Pure resistor:} \quad V = IR$$

$$\text{Pure inductor:} \quad V = IX_L$$

where $X_L = 2\pi fL$ is called the *inductive reactance*. Its unit is ohms when L is in henries and f is in hertz.

$$\text{Pure capacitor:} \quad V = IX_C$$

where $X_C = 1/2\pi fC$ is called the *capacitive reactance*. Its unit is ohms when C is in farads.

PHASE: When an ac voltage is applied to a pure resistance, the voltage across the resistance and the current through it attain their maximum values at the same instant and their zero values at the same instant; the voltage and current are said to be *in-phase*.

When an ac voltage is applied to a pure inductance, the voltage across the inductance reaches its maximum value one-quarter cycle ahead of the current, i.e., when the current is zero. The back emf of the inductance causes the current through the inductance to lag behind the voltage by one-quarter cycle (or 90°), and the two are 90° *out-of-phase*.

When an ac voltage is applied to a pure capacitor, the voltage across it lags 90° behind the current flowing through it. Current must flow before the voltage across (and charge on) the capacitor can build up.

In more complicated situations involving combinations of R, L, and C, the voltage and current are usually (but not always) out-of-phase. The angle by which the voltage lags or leads the current is called the *phase angle*.

THE IMPEDANCE (Z) of a series circuit containing resistance, inductance, and capacitance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

with Z in ohms. If a voltage V is applied to such a series circuit, then a form of Ohm's Law relates V to the current I through it:

$$V = IZ$$

The phase angle ϕ between V and I is given by

$$\tan \phi = \frac{X_L - X_C}{R} \qquad \text{or} \qquad \cos \phi = \frac{R}{Z}$$

POWER LOSS: Suppose that an ac voltage V is impressed across an impedance of any type. It gives rise to a current I through the impedance, and the phase angle between V and I is ϕ. The power loss in the impedance is given by

$$\text{Power loss} = VI \cos \phi$$

The quantity $\cos \phi$ is called the *power factor*. It is unity for a pure resistor; but it is zero for a pure inductor or capacitor (no power loss occurs in a pure inductor or capacitor).

Solved Problems

A sinusoidal, 60.0-Hz, ac voltage is read to be 120 V by an ordinary voltmeter. (*a*) What is the maximum value the voltage takes on during a cycle? (*b*) What is the equation for the voltage?

(*a*) $$V = \frac{v_0}{\sqrt{2}} \qquad \text{or} \qquad v_0 = \sqrt{2}V = \sqrt{2}(120\ \text{V}) = 170\ \text{V}$$

(*b*) $$v = v_0 \sin 2\pi f t = (170\ \text{V}) \sin 120\pi t$$

where t is in s.

A voltage $v = (60.0\ \text{V}) \sin 120\pi t$ is applied across a 20.0-Ω resistor. What will an ac ammeter in series with the resistor read?

The rms voltage across the resistor is

$$V = 0.707 v_0 = (0.707)(60.0\ \text{V}) = 42.4\ \text{V}$$

Then $$I = \frac{V}{R} = \frac{42.4\ \text{V}}{20.0\ \Omega} = 2.12\ \text{A}$$

A 120-V ac voltage source is connected across a 2.0-μF capacitor. Find the current to the capacitor if the frequency of the source is (*a*) 60 Hz and (*b*) 60 kHz. (*c*) What is the power loss in the capacitor?

(*a*)
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\ \text{s}^{-1})(2.0 \times 10^{-6}\ \text{F})} = 1.33\ \text{k}\Omega$$

Then
$$I = \frac{V}{X_C} = \frac{120\ \text{V}}{1330\ \Omega} = 0.090\ \text{A}$$

(*b*) Now $X_C = 1.33\ \Omega$, so $I = 90$ A. Notice that the impedance effect of a capacitor varies inversely with the frequency.

(*c*)
$$\text{Power loss} = VI\cos\phi = VI\cos 90^\circ = 0$$

A 120-V ac voltage source is connected across a pure 0.700-H inductor. Find the current through the inductor if the frequency of the source is (*a*) 60.0 Hz and (*b*) 60.0 kHz. (*c*) What is the power loss in the inductor?

(*a*)
$$X_L = 2\pi fL = 2\pi(60.0\ \text{s}^{-1})(0.700\ \text{H}) = 264\ \Omega$$

Then
$$I = \frac{V}{X_L} = \frac{120\ \text{V}}{264\ \Omega} = 0.455\ \text{A}$$

(*b*) Now $X_L = 264 \times 10^3\ \Omega$, so $I = 0.455 \times 10^{-3}$ A. Notice that the impedance effect of an inductor varies directly with the frequency.

(*c*)
$$\text{Power loss} = VI\cos\phi = VI\cos 90^\circ = 0$$

A coil having inductance 0.14 H and resistance of 12 Ω is connected across a 110-V, 25-Hz line. Compute (*a*) the current in the coil, (*b*) the phase angle between the current and the supply voltage, (*c*) the power factor, and (*d*) the power loss in the coil.

(*a*)
$$X_L = 2\pi fL = 2\pi(25)(0.14) = 22.0\ \Omega$$

and
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (22 - 0)^2} = 25.1\ \Omega$$

so
$$I = \frac{V}{Z} = \frac{110\ \text{V}}{25.1\ \Omega} = 4.4\ \text{A}$$

(*b*)
$$\tan\phi = \frac{X_L - X_C}{R} = \frac{22 - 0}{12} = 1.83 \qquad \text{or} \qquad \phi = 61.3°$$

The voltage leads the current by 61°.

(*c*)
$$\text{Power factor} = \cos\phi = \cos 61.3° = 0.48$$

(*d*)
$$\text{Power loss} = VI\cos\phi = (110\ \text{V})(4.4\ \text{A})(0.48) = 0.23\ \text{kW}$$

Or, since power loss occurs only because of the resistance of the coil,

$$\text{Power loss} = I^2R = (4.4\ \text{A})^2(12\ \Omega) = 0.23\ \text{kW}$$

Solved Problems

A voltmeter reads 80.0 V when it is connected across the terminals of a sinusoidal power source with $f = 1000$ Hz. Write the equation for the instantaneous voltage provided by the source.

A coil has resistance 20 Ω and inductance 0.35 H. Compute its reactance and its impedance to an alternating current of 25 cycles/s.

A 10.0-μF capacitor is in series with a 40.0-Ω resistance, and the combination is connected to a 110-V, 60.0-Hz line. Calculate (*a*) the capacitive reactance, (*b*) the impedance of the circuit, (*c*) the current in the circuit, (*d*) the phase angle between current and supply voltage, and (*e*) the power factor for the circuit.

A circuit having a resistance, an inductance, and a capacitance in series is connected to a 110-V ac line. For the circuit, $R = 9.0\ \Omega$, $X_L = 28\ \Omega$, and $X_C = 16\ \Omega$. Compute (*a*) the impedance of the circuit, (*b*) the current, (*c*) the phase angle between the current and the supply voltage, and (*d*) the power factor of the circuit.