University of Science and Technology

Faculty of Computer Science and Information Technology

Course Handbook

Physics II

Course Coordinator

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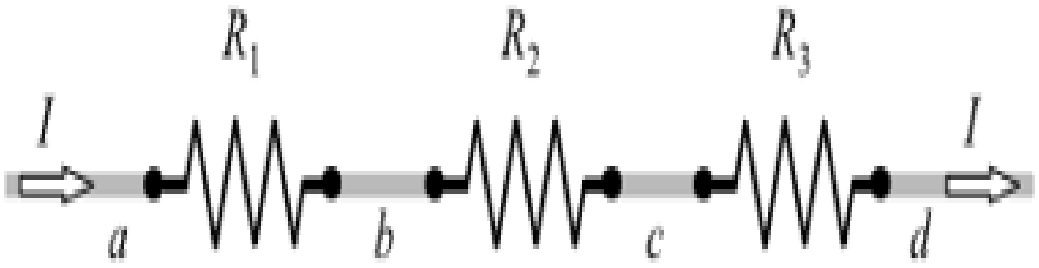
Lecture Four

Equivalent Resistance; Simple Circuits

Equivalent Resistance; Simple Circuits

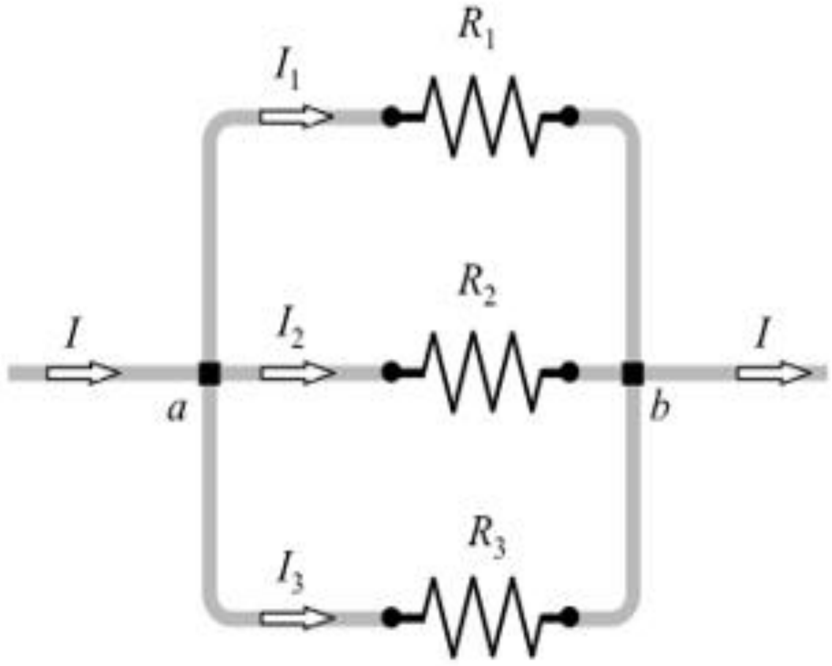
RESISTORS IN SERIES: When current can follow only one path as it flows through two or more resistors connected in line, the resistors are in *series*. In other words, when one and only one terminal of a resistor is connected directly to one and only one terminal of another resistor, the two are in series and the same current passes through both. A *node* is a point where three or more current-carrying wires or branches meet. There are no nodes between circuit elements (such as capacitors, resistors, and batteries) that are connected in series. A typical case is shown in Fig. 28-1(*a*). For several resistors in series, their equivalent resistance R_{eq} is given by where $R_1, R_2, R_3, \ldots$, are the resistances of the several resistors. Observe that resistances in series combine like capacitances in parallel (see Chapter 25). It is assumed that all connection wire is effectively resistanceless.

In a series combination, the current through each resistance is the same as that through all the others. The potential drop (p.d.) across the combination is equal to the sum of the individual potential drops. *The equivalent resistance in series is always greater than the largest of the individual resistances.*



RESISTORS IN PARALLEL: Several resistors are connected in *parallel* between two nodes if one end of each resistor is connected to one node and the other end of each is connected to the other node. A typical case is shown in Fig. 28-1(b), where points a and b are nodes. Their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad \text{(parallel combination)}$$



The equivalent resistance in parallel is always less than the smallest of the individual resistances. Connecting additional resistances in parallel decreases R_{eq} for the combination. Observe that resistances in parallel combine like capacitances in series (see Chapter 25).

The potential drop V across each resistor in a parallel combination is the same as the potential drop across each of the others. The current through the nth resistor is $I_n = V/R_n$ and the total current entering the combination is equal to the sum of the individual branch currents.

Solved Problems

Derive the formula for the equivalent resistance R_{eq} of resistors R_1, R_2, and R_3 (*a*) in series and (*b*) in parallel, as shown in Fig. 28-1(*a*) and (*b*).

(*a*) For the series network,

$$V_{ad} = V_{ab} + V_{bc} + V_{cd} = IR_1 + IR_2 + IR_3$$

since the current I is the same in all three resistors. Dividing by I gives

$$\frac{V_{ad}}{I} = R_1 + R_2 + R_3 \qquad \text{or} \qquad R_{eq} = R_1 + R_2 + R_3$$

since V_{ad}/I is by definition the equivalent resistance R_{eq} of the network.

(*b*) The p.d. is the same for all three resistors, whence

$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

Since the line current I is the sum of the branch currents,

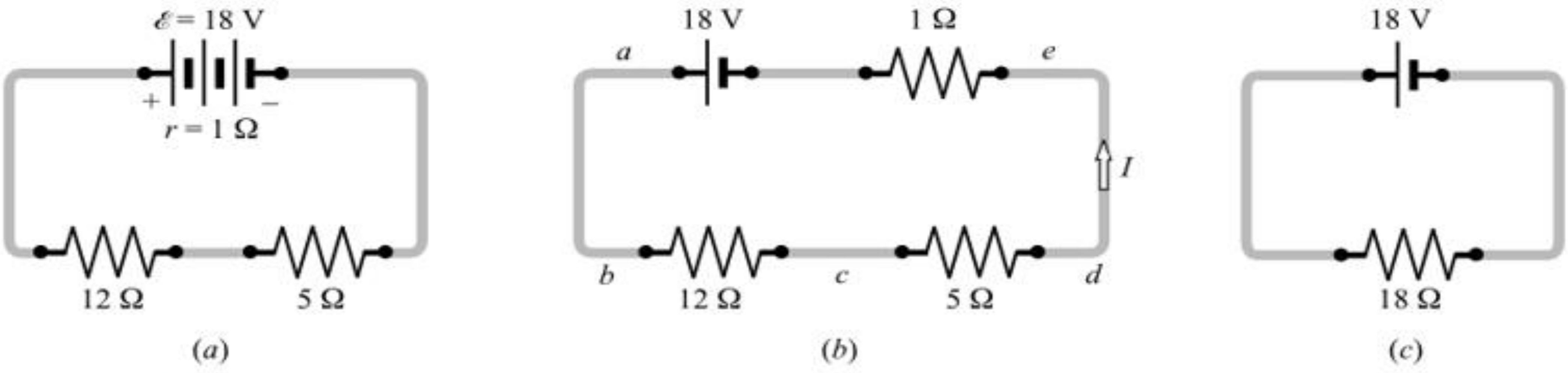
$$I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

Dividing by V_{ab} gives

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{or} \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

since V_{ab}/I is by definition the equivalent resistance R_{eq} of the network.

As shown in Fig. 28-2(a), a battery (internal resistance 1 Ω) is connected in series with two resistors. Compute (a) the current in the circuit, (b) the p.d. across each resistor, and (c) the terminal p.d. of the battery.



$$R_{eq} = 5\ \Omega + 12\ \Omega + 1\ \Omega = 18\ \Omega$$

$$I = \frac{V}{R} = \frac{18\ \text{V}}{18\ \Omega} = 1.0\ \text{A}$$

$$V_{bc} = IR_{bc} = (1.0\ \text{A})(12\ \Omega) = 12\ \text{V}$$

$$V_{cd} = IR_{cd} = (1.0\ \text{A})(5\ \Omega) = 5\ \text{V}$$

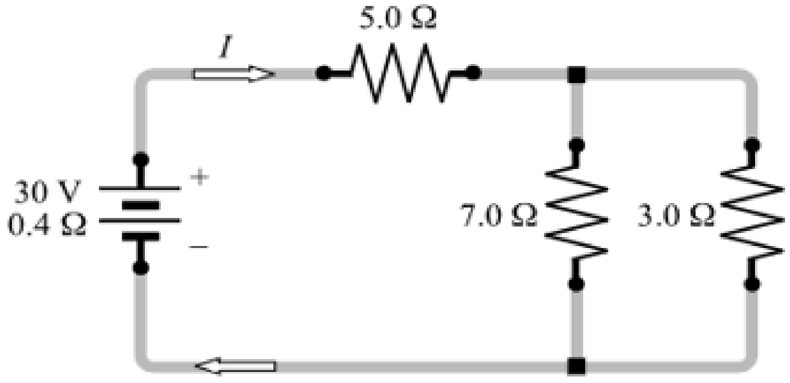
What resistance must be placed in parallel with 12 Ω to obtain a combined resistance of 4 Ω?

From $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

we have $$\frac{1}{4\ \Omega} = \frac{1}{12\ \Omega} + \frac{1}{R_2}$$

so $$R_2 = 6\ \Omega$$

For each circuit shown in Fig. 28-3, determine the current I through the battery.

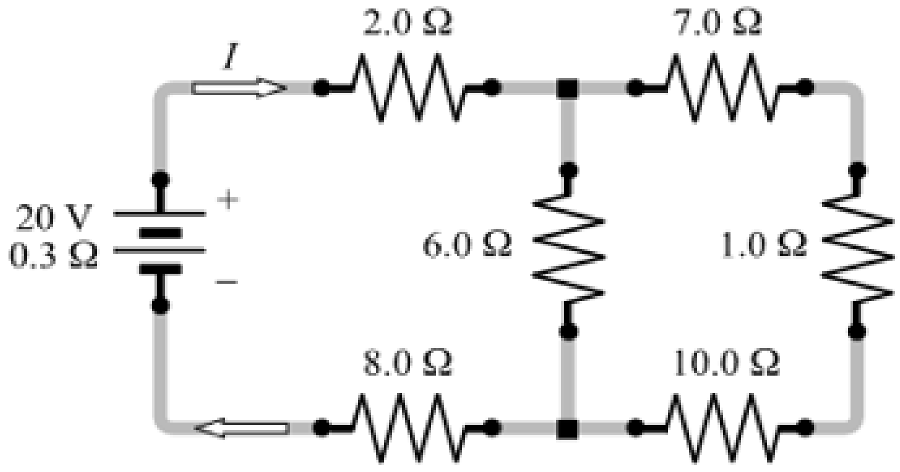


(a)

$$\frac{1}{R_1} = \frac{1}{3.0\ \Omega} + \frac{1}{7.0\ \Omega} = \frac{10}{21\ \Omega} \qquad \text{or} \qquad R_1 = 2.1\ \Omega$$

$$R_{eq} = 2.1\ \Omega + 5.0\ \Omega + 0.4\ \Omega = 7.5\ \Omega$$

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{30\ \text{V}}{7.5\ \Omega} = 4.0\ \text{A}$$



(*b*)

(*b*) The 7.0-Ω, 1.0-Ω, and 10.0-Ω resistors are in series; their joint resistance is 18.0 Ω. Then 18.0 Ω is in parallel with 6.0 Ω; their combined resistance R_1 is given by

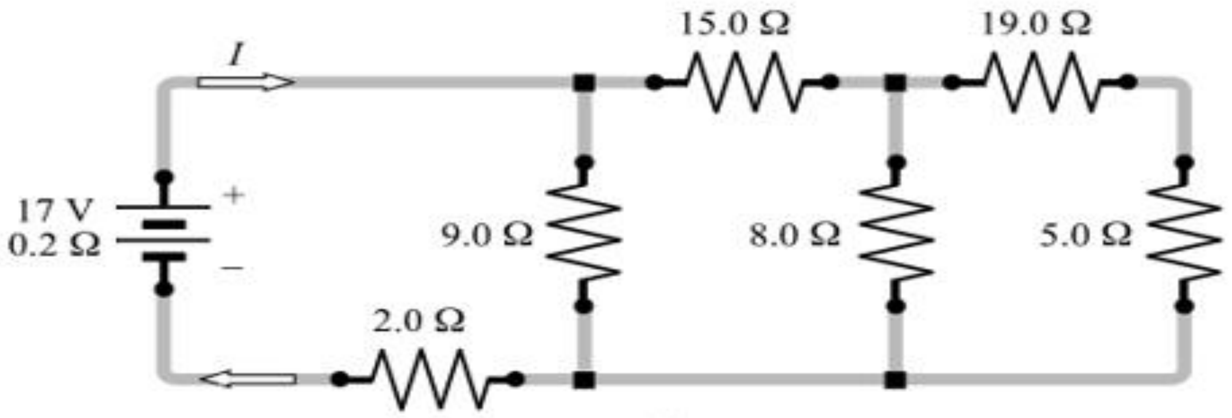
$$\frac{1}{R_1} = \frac{1}{18.0\ \Omega} + \frac{1}{6.0\ \Omega} \qquad \text{or} \qquad R_1 = 4.5\ \Omega$$

Hence, the equivalent resistance of the entire circuit is

$$R_{eq} = 4.5\ \Omega + 2.0\ \Omega + 8.0\ \Omega + 0.3\ \Omega = 14.8\ \Omega$$

and the battery current is

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{20\ \text{V}}{14.8\ \Omega} = 1.4\ \text{A}$$



(c)

(c) The 5.0-Ω and 19.0-Ω resistors are in series; their joint resistance is 24.0 Ω. Then 24.0 Ω is in parallel with 8.0 Ω; their joint resistance R_1 is given by

$$\frac{1}{R_1} = \frac{1}{24.0\ \Omega} + \frac{1}{8.0\ \Omega} \qquad \text{or} \qquad R_1 = 6.0\ \Omega$$

Now $R_1 = 6.0\ \Omega$ is in series with 15.0 Ω; their joint resistance is $6.0\ \Omega + 15.0\ \Omega = 21.0\ \Omega$. Thus 21.0 Ω is in parallel with 9.0 Ω; their combined resistance is found from

$$\frac{1}{R_2} = \frac{1}{21.0\ \Omega} + \frac{1}{9.0\ \Omega} \qquad \text{or} \qquad R_2 = 6.3\ \Omega$$

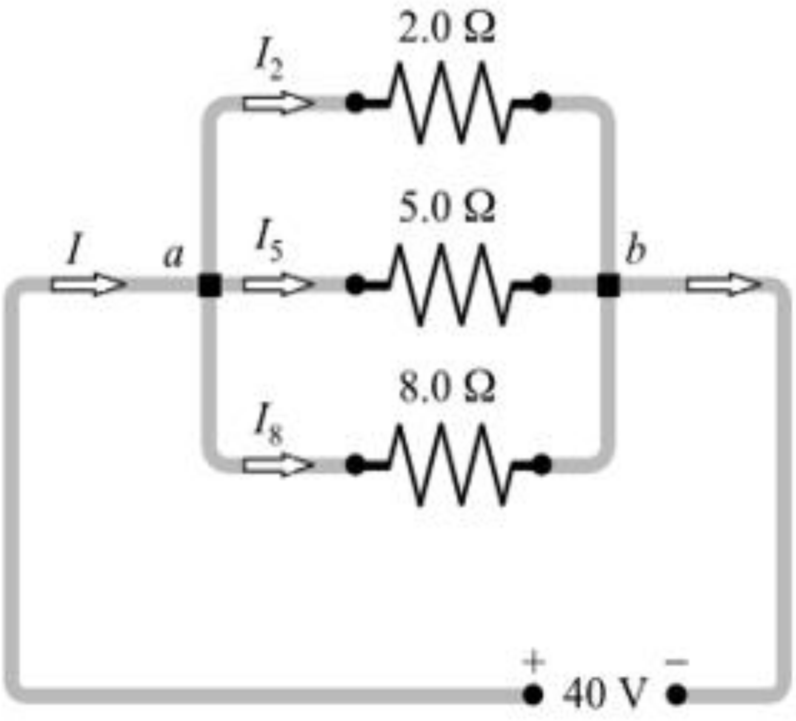
Hence the equivalent resistance of the entire circuit is

$$R_{eq} = 6.3\ \Omega + 2.0\ \Omega + 0.2\ \Omega = 8.5\ \Omega$$

and the battery current is

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{17\ \text{V}}{8.5\ \Omega} = 2.0\ \text{A}$$

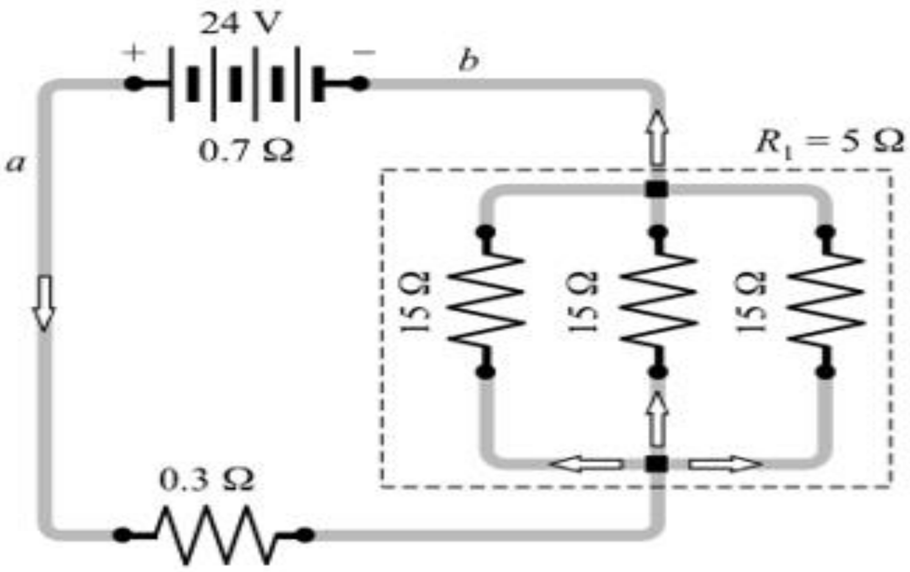
For the circuit shown in Fig. 28-4, find the current in each resistor and the current drawn from the battery.



$$I_2 = \frac{40\ \text{V}}{2.0\ \Omega} = 20\ \text{A} \qquad I_5 = \frac{40\ \text{V}}{5.0\ \Omega} = 8.0\ \text{A} \qquad I_8 = \frac{40\ \text{V}}{8.0\ \Omega} = 5.0\ \text{A}$$

$$I = I_2 + I_5 + I_8 = 20\ \text{A} + 8.0\ \text{A} + 5.0\ \text{A} = 33\ \text{A}$$

In Fig. 28-5, the battery has an internal resistance of 0.7 Ω. Find (*a*) the current drawn from the battery, (*b*) the current in each 15-Ω resistor, and (*c*) the terminal voltage of the battery.



(*a*) For parallel group resistance R_1 we have

$$\frac{1}{R_1} = \frac{1}{15\ \Omega} + \frac{1}{15\ \Omega} + \frac{1}{15\ \Omega} = \frac{3}{15\ \Omega} \qquad \text{or} \qquad R_1 = 5.0\ \Omega$$

Then $$R_{eq} = 5.0\ \Omega + 0.3\ \Omega + 0.7\ \Omega = 6.0\ \Omega$$

and $$I = \frac{\mathscr{E}}{R_{eq}} = \frac{24\ \text{V}}{6.0\ \Omega} = 4.0\ \text{A}$$

The three-resistor combination is equivalent to $R_1 = 5.0\ \Omega$. A current of 4.0 A flows through it. Hence, the p.d. across the combination is

$$IR_1 = (4.0\ \text{A})(5.0\ \Omega) = 20\ \text{V}$$

This is also the p.d. across each 15-Ω resistor. Therefore, the current through each 15-Ω resistor is

$$I_{15} = \frac{V}{R} = \frac{20\ \text{V}}{15\ \Omega} = 1.3\ \text{A}$$

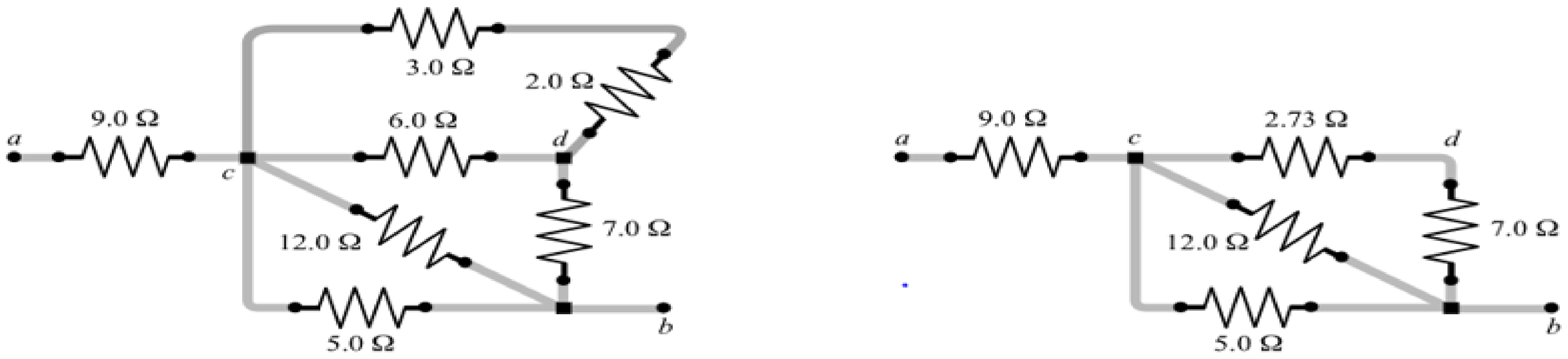
(*c*) We start at *a* and go to *b* outside the battery:

$$V \text{ from } a \text{ to } b = -(4.0\ \text{A})(0.3\ \Omega) - (4.0\ \text{A})(5.0\ \Omega) = -21.2\ \text{V}$$

The terminal p.d. of the battery is 21.2 V. Or, we could write for this case of a discharging battery,

$$\text{Terminal p.d.} = \mathscr{E} - Ir = 24\ \text{V} - (4.0\ \text{A})(0.7\ \Omega) = 21.2\ \text{V}$$

Find the equivalent resistance between points a and b for the combination shown in Fig. 28-6(a).



(a) (b)

The 3.0-Ω and 2.0-Ω resistors are in series and are equivalent to a 5.0-Ω resistor. The equivalent 5.0 Ω is in parallel with the 6.0 Ω, and their equivalent, R_1, is

$$\frac{1}{R_1} = \frac{1}{5.0\ \Omega} + \frac{1}{6.0\ \Omega} = 0.20 + 0.167 = 0.367\ \Omega^{-1} \qquad \text{or} \qquad R_1 = 2.73\ \Omega$$

The circuit thus far reduced is shown in Fig. 28-6(b).

The 7.0 Ω and 2.73 Ω are equivalent to 9.73 Ω. Now the 5.0 Ω, 12.0 Ω, and 9.73 Ω are in parallel, and their equivalent, R_2, is

$$\frac{1}{R_2} = \frac{1}{5.0\ \Omega} + \frac{1}{12.0\ \Omega} + \frac{1}{9.73\ \Omega} = 0.386\ \Omega^{-1} \qquad \text{or} \qquad R_2 = 2.6\ \Omega$$

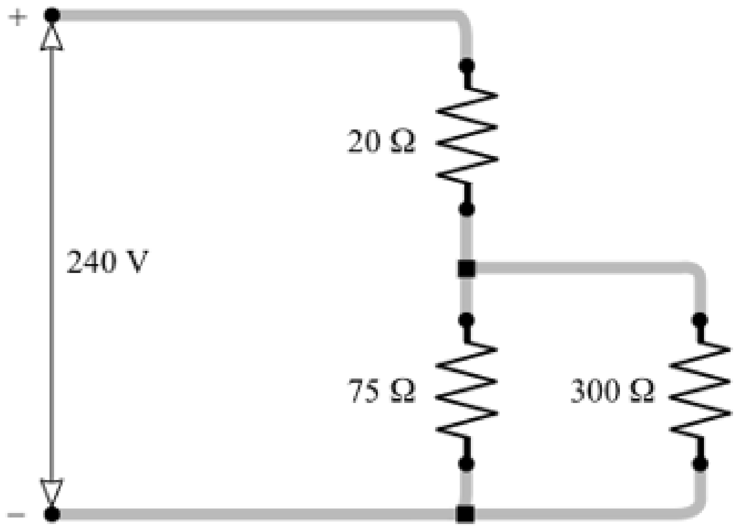
This 2.6 Ω is in series with the 9.0-Ω resistor. Therefore, the equivalent resistance of the combination is 9.0 Ω + 2.6 Ω = 11.6 Ω.

Supplementary Problems

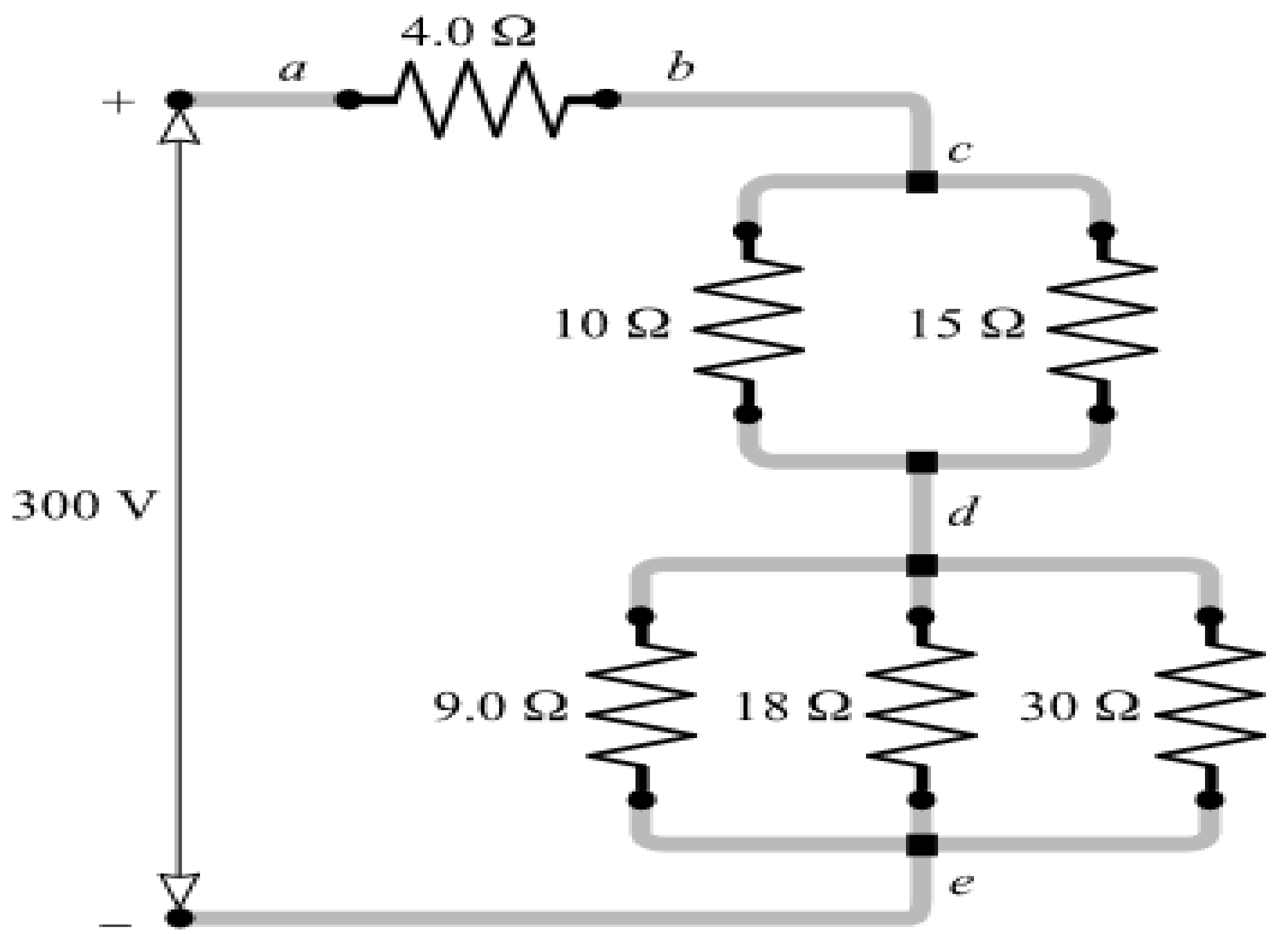
Compute the equivalent resistance of 4.0 Ω and 8.0 Ω (*a*) in series and (*b*) in parallel. *Ans.* (*a*) 12 Ω; (*b*) 2.7 Ω

Compute the equivalent resistance of (*a*) 3.0 Ω, 6.0 Ω, and 9.0 Ω in parallel; (*b*) 3.0 Ω, 4.0 Ω, 7.0 Ω, 10.0 Ω, and 12.0 Ω in parallel; (*c*) three 33-Ω heating elements in parallel; (*d*) twenty 100-Ω lamps in parallel. *Ans.* (*a*) 1.6 Ω; (*b*) 1.1 Ω; (*c*) 11 Ω; (*d*) 5.0 Ω

For the circuit shown in Fig. 28-13, find the current through each resistor and the potential drop across each resistor. *Ans.* for 20 Ω, 3.0 A and 60 V; for 75 Ω, 2.4 A and 180 V; for 300 Ω, 0.6 A and 180 V



For the circuit shown in Fig. 28-14, find (*a*) its equivalent resistance; (*b*) the current drawn from the power source; (*c*) the potential differences across *ab*, *cd*, and *de*; (*d*) the current in each resistor. *Ans.* (*a*) 15 Ω; (*b*) 20 A; (*c*) $V_{ab} = 80$ V, $V_{cd} = 120$ V, $V_{de} = 100$ V; (*d*) $I_4 = 20$ A, $I_{10} = 12$ A, $I_{15} = 8$ A, $I_9 = 11.1$ A, $I_{18} = 5.6$ A, $I_{30} = 3.3$ A


4.0 Ω
a
b
+
c
10 Ω
15 Ω
300 V
d
9.0 Ω
18 Ω
30 Ω
e
−