University of Science and Technology

Faculty of Computer Science and Information Technology

Course Handbook

Physics II

Course Coordinator

Lecturer: Dr. Alzubair Jaafer

Lecture two

Potential; Capacitance

Potential; Capacitance

THE POTENTIAL DIFFERENCE between point A and point B is the work done against electrical forces in carrying a *unit* positive test-charge from A to B. We represent the potential difference between A and B by $V_B - V_A$ or by V. Its units are those of work per charge (joules/coulomb) and are called *volts* (V):

$$1\ \text{V} = 1\ \text{J/C}$$

Because work is a scalar quantity, so too is potential difference. Like work, potential difference may be positive or negative.

The work W done in transporting a charge q from one point A to a second point B is

$$W = q(V_B - V_A) = qV$$

where the appropriate sign (+ or −) must be given to the charge. If both $(V_B - V_A)$ and q are positive (or negative), the work done is positive. If $(V_B - V_A)$ and q have opposite signs, the work done is negative.

ABSOLUTE POTENTIAL: The absolute potential at a point is the work done against electric forces in carrying a unit positive test-charge from infinity to that point. Hence the absolute potential at a point B is the difference in potential from $A = \infty$ to B.

Consider a point charge q in vacuum and a point P at a distance r from the point charge. The absolute potential at P due to the charge q is

$$V = k\frac{q}{r}$$

where $k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$ is the Coulomb constant. The absolute potential at infinity (at $r = \infty$) is zero.

Because of the superposition principle and the scalar nature of potential difference, the absolute potential at a point due to a number of point charges is

$$V = k\sum\frac{q_i}{r_i}$$

where the r_i are the distances of the charges q_i from the point in question. Negative q's contribute negative terms to the potential, while positive q's contribute positive terms.

The absolute potential due to a uniformly charged sphere, at points *outside* the sphere or *on* its surface is $V = kq/r$, where q is the charge on the sphere. This potential is the same as that due to a point charge q placed at the position of the sphere's center.

ELECTRICAL POTENTIAL ENERGY (PE_E)**:** To carry a charge q from infinity to a point where the absolute potential is V, work in the amount qV must be done on the charge. This work appears as electrical potential energy (PE_E) stored in the charge.

Similarly, when a charge q is carried through a *potential difference* V, work in the amount qV must be done on the charge. This work results in a change qV in the PE_E of the charge. For a potential *rise*, V will be positive and the PE_E will increase if q is positive. But for a potential *drop*, V will be negative and the PE_E of the charge will decrease if q is positive.

V RELATED TO E: Suppose that in a certain region the electric field is uniform and is in the x-direction. Call its magnitude E_x. Because E_x is the force on a unit positive test-charge, the work done in moving the test-charge through a distance x is (from $W = F_x x$)

$$V = E_x x$$

The field between two large, parallel, oppositely charged metal plates is uniform. We can therefore use this equation to relate the electric field E between the plates to the plate separation d and their potential difference V: For parallel plates,

$$V = Ed$$

ELECTRON VOLT ENERGY UNIT: The work done in carrying a charge $+e$ (coulombs) through a potential rise of exactly 1 volt is defined to be 1 *electron volt* (eV). Therefore,

$$1 \text{ eV} = (1.602 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.602 \times 10^{-19} \text{ J}$$

Equivalently,

$$\text{Work or energy (in eV)} = \frac{\text{work (in joules)}}{e}$$

A CAPACITOR is a device that stores charge. Often, although certainly not always, it consists of two conductors separated by an insulator or dielectric. The *capacitance* (C) of a capacitor is defined as

$$\text{Capacitance} = \frac{\text{magnitude of charge on either conductor}}{\text{magnitude of potential difference between conductors}}$$

For q in coulombs and V in volts, C is in *farads* (F).

PARALLEL-PLATE CAPACITOR: The capacitance of a parallel-plate capacitor whose opposing plate faces, each of area A, are separated by a small distance d is given by

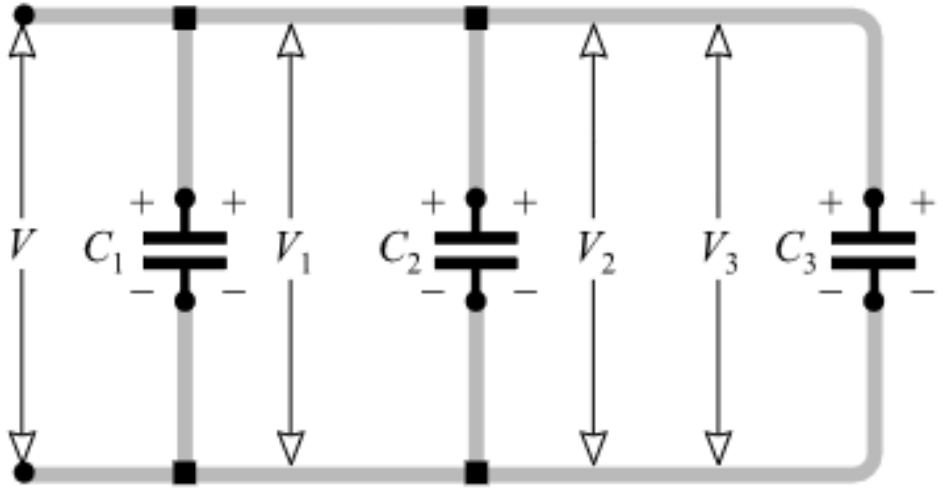
$$C = K\epsilon_0 \frac{A}{d}$$

where $K = \epsilon/\epsilon_0$ is the dimensionless dielectric constant (see Chapter 24) of the nonconducting material (the *dielectric*) between the plates, and

$$\epsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2 = 8.85 \times 10^{-12}\ \text{F/m}$$

For vacuum, $K = 1$, so that a dielectric-filled parallel-plate capacitor has a capacitance K times larger than the same capacitor with vacuum between its plates. This result holds for a capacitor of arbitrary shape.

CAPACITORS IN PARALLEL AND SERIES: As shown in Fig. 25-1, capacitances add for capacitors in parallel, whereas reciprocal capacitances add for capacitors in series.

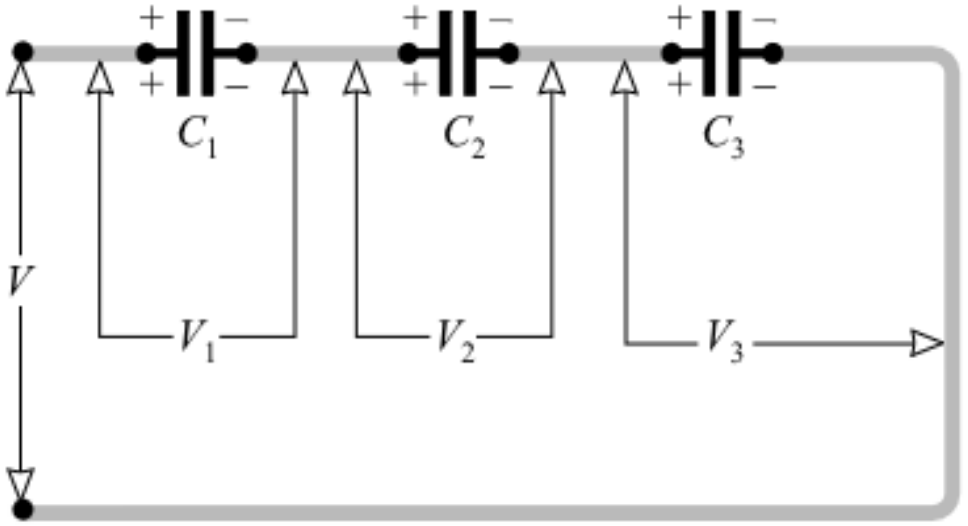


$$q = q_1 + q_2 + q_3$$

$$V = V_1 = V_2 = V_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

(*a*) Capacitors in parallel



$$q = q_1 = q_2 = q_3$$

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

(*b*) Capacitors in series

Fig. 25-1

ENERGY STORED IN A CAPACITOR: The energy (PE_E) stored in a capacitor of capacitance C that has a charge q and a potential difference V is

$$PE_E = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$$

Solved Problems

How much work is required to carry an electron from the positive terminal of a 12-V battery to the negative terminal?

$$W = qV = (-1.6 \times 10^{-19}\,\text{C})(-12\,\text{V}) = 1.9 \times 10^{-18}\,\text{J}$$

How much electrical potential energy does a proton lose as it falls through a potential drop of 5 kV?

$$\text{Change in } PE_E = Vq = (-5 \times 10^3\,\text{V})(1.6 \times 10^{-19}\,\text{C}) = -8 \times 10^{-16}\,\text{J}$$

(*a*) What is the absolute potential at each of the following distances from a charge of 2.0 μC: $r = 10$ cm and $r = 50$ cm? (*b*) How much work is required to carry a 0.05 μC charge from the point at $r = 50$ cm to that at $r = 10$ cm?

(*a*)
$$V_{10} = k\frac{q}{r} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.0 \times 10^{-6}\ \text{C}}{0.10\ \text{m}} = 1.8 \times 10^5\ \text{V}$$

$$V_{50} = \frac{10}{50}V_{10} = 36\ \text{kV}$$

(*b*)
$$\text{Work} = q(V_{10} - V_{50}) = (5 \times 10^{-8}\ \text{C})(1.44 \times 10^5\ \text{V}) = 7.2\ \text{mJ}$$

The following point charges are placed on the x-axis: $+2.0\,\mu C$ at $x = 20$ cm, $-3.0\,\mu C$ at $x = 30$ cm, $-4.0\,\mu C$ at $x = 40$ cm. Find the absolute potential on the axis at $x = 0$.

$$V = k\sum \frac{q_i}{r_i} = (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\left(\frac{2.0 \times 10^{-6}\ \mathrm{C}}{0.20\ \mathrm{m}} + \frac{-3.0 \times 10^{-6}\ \mathrm{C}}{0.30\ \mathrm{m}} + \frac{-4.0 \times 10^{-6}\ \mathrm{C}}{0.40\ \mathrm{m}}\right)$$

$$= (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})(10 \times 10^{-6}\ \mathrm{C/m} - 10 \times 10^{-6}\ \mathrm{C/m} - 10 \times 10^{-6}\ \mathrm{C/m}) = -90\ \mathrm{kV}$$

In Fig. 25-2, the potential difference between the metal plates is 40 V. (*a*) Which plate is at the higher potential? (*b*) How much work must be done to carry a +3.0 C charge from *B* to *A*? From *A* to *B*? (*c*) How do we know that the electric field is in the direction indicated? (*d*) If the plate separation is 5.0 mm, what is the magnitude of $\vec{\mathbf{E}}$?

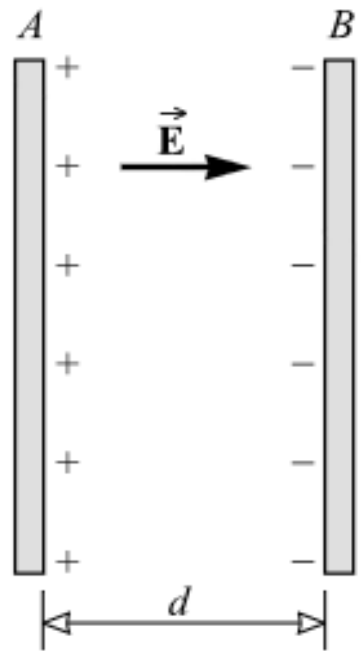


Fig. 25-2

(*a*) A positive test charge between the plates is repelled by *A* and attracted by *B*. Left to itself, the positive test charge will move from *A* to *B*, and so *A* is at the higher potential.

(*b*) The magnitude of the work done in carrying a charge *q* through a potential difference *V* is *qV*. Thus the magnitude of the work done in the present situation is

$$W = (3.0\ \text{C})(40\ \text{V}) = 0.12\ \text{kJ}$$

Because a positive charge between the plates is repelled by *A*, positive work (+120 J) must be done to drag the +3.0 C charge from *B* to *A*. To restrain the charge as it moves from *A* to *B*, negative work (−120 J) is done.

(*c*) A positive test-charge between the plates experiences a force directed from *A* to *B* and this is, by definition, the direction of the field.

(*d*) For parallel plates, $V = Ed$. Therefore,

$$E = \frac{V}{d} = \frac{40\ \text{V}}{0.005\,0\ \text{m}} = 8.0\ \text{kV/m}$$

Notice that the SI units for electric field, V/m and N/C, are identical.

A capacitor has a capacitance of 8.0 μF with air between its plates. Determine its capacitance when a dielectric with dielectric constant 6.0 is placed between its plates.

$$C \text{ with dielectric} = K(C \text{ with air}) = (6.0)(8.0\ \mu\text{F}) = 48\ \mu\text{F}$$

What is the charge on a 300 pF capacitor when it is charged to a voltage of 1.0 kV?

$$q = CV = (300 \times 10^{-12}\ \text{F})(1000\ \text{V}) = 3.0 \times 10^{-7}\ \text{C} = 0.30\ \mu\text{C}$$

A metal sphere mounted on an insulating rod carries a charge of 6.0 nC when its potential is 200 V higher than its surroundings. What is the capacitance of the capacitor formed by the sphere and its surroundings?

$$C = \frac{q}{V} = \frac{6.0 \times 10^{-9}\ \text{C}}{200\ \text{V}} = 30\ \text{pF}$$

The series combination of two capacitors shown in Fig. 25-6 is connected across 1000 V. Compute (*a*) the equivalent capacitance C_{eq} of the combination, (*b*) the magnitudes of the charges on the capacitors, (*c*) the potential differences across the capacitors, and (*d*) the energy stored in the capacitors.

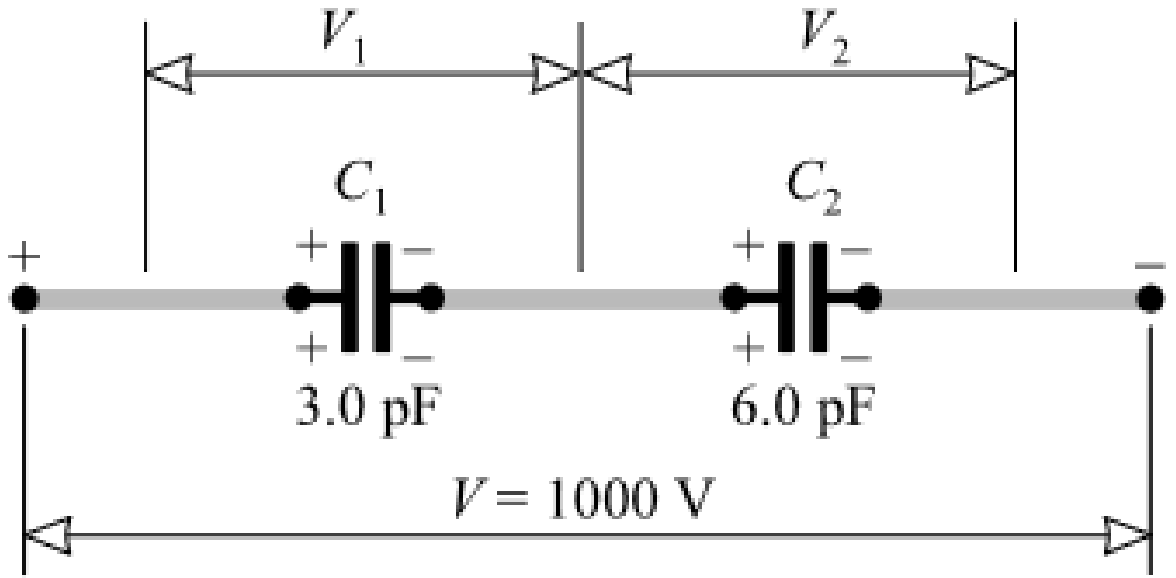


Fig. 25-6

(*a*)
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.0\ \text{pF}} + \frac{1}{6.0\ \text{pF}} = \frac{1}{2.0\ \text{pF}}$$

from which $C = 2.0$ pF.

(*b*) In a series combination, each capacitor carries the same charge, which is the charge on the combination. Thus, using the result of (*a*), we have

$$q_1 = q_2 = q = C_{eq}V = (2.0 \times 10^{-12}\text{F})(1000\ \text{V}) = 2.0\ \text{nC}$$

(*c*)
$$V_1 = \frac{q_1}{C_1} = \frac{2.0 \times 10^{-9}\ \text{C}}{3.0 \times 10^{-12}\ \text{F}} = 667\ \text{V} = 0.67\ \text{kV}$$

$$V_2 = \frac{q_2}{C_2} = \frac{2.0 \times 10^{-9}\ \text{C}}{6.0 \times 10^{-12}\ \text{F}} = 333\ \text{V} = 0.33\ \text{kV}$$

(*d*)
$$\text{Energy in } C_1 = \tfrac{1}{2}q_1V_1 = \tfrac{1}{2}(2.0 \times 10^{-9}\ \text{C})(667\ \text{V}) = 6.7 \times 10^{-7}\ \text{J} = 0.67\ \mu\text{J}$$

$$\text{Energy in } C_2 = \tfrac{1}{2}q_2V_2 = \tfrac{1}{2}(2.0 \times 10^{-9}\ \text{C})(333\ \text{V}) = 3.3 \times 10^{-7}\ \text{J} = 0.33\ \mu\text{J}$$

$$\text{Energy in combination} = (6.7 + 3.3) \times 10^{-7}\ \text{J} = 10 \times 10^{-7}\ \text{J} = 1.0\ \mu\text{J}$$

The last result is also directly given by $\frac{1}{2}qV$ or $\frac{1}{2}C_{eq}V^2$.

The parallel capacitor combination shown in Fig. 25-7 is connected across a 120 V source. Determine the equivalent capacitance C_{eq}, the charge on each capacitor, and the charge on the combination.

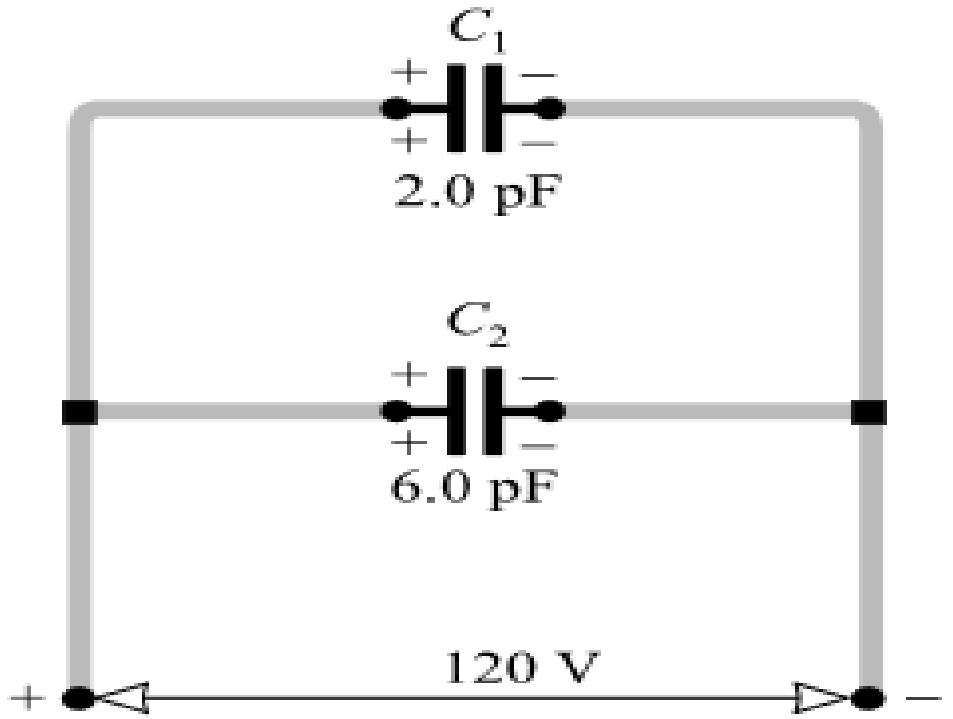


Fig. 25–7

For a parallel combination,

$$C_{eq} = C_1 + C_2 = 2.0\ \text{pF} + 6.0\ \text{pF} = 8.0\ \text{pF}$$

Each capacitor has a 120 V potential difference impressed on it. Therefore,

$$q_1 = C_1 V_1 = (2.0 \times 10^{-12}\ \text{F})(120\ \text{V}) = 0.24\ \text{nC}$$

$$q_2 = C_2 V_2 = (6.0 \times 10^{-12}\ \text{F})(120\ \text{V}) = 0.72\ \text{nC}$$

The charge on the combination is $q_1 + q_2 = 960$ pC. Or, we could write

$$q = C_{eq} V = (8.0 \times 10^{-12}\ \text{F})(120\ \text{V}) = 0.96\ \text{nC}$$

A 1.2 μF capacitor is charged to 3.0 kV. Compute the energy stored in the capacitor.

$$\text{Energy} = \tfrac{1}{2}aV = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(1.2 \times 10^{-6}\ \text{F})(3000\ \text{V})^2 = 5.4\ \text{J}$$

A certain parallel-plate capacitor consists of two plates, each with area 200 cm^2, separated by a 0.40-cm air gap. (*a*) Compute its capacitance. (*b*) If the capacitor is connected across a 500 V source, find the charge on it, the energy stored in it, and the value of E between the plates. (*c*) If a liquid with $K = 2.60$ is poured between the plates so as to fill the air gap, how much additional charge will flow onto the capacitor from the 500 V source?

(*a*) For a parallel-plate capacitor with air gap,

$$C = K\epsilon_0 \frac{A}{d} = (1)(8.85 \times 10^{-12}\ \text{F/m}) \frac{200 \times 10^{-4}\ \text{m}^2}{4.0 \times 10^{-3}\ \text{m}} = 4.4 \times 10^{-11}\ \text{F} = 44\ \text{pF}$$

(*b*)
$$q = CV = (4.4 \times 10^{-11}\ \text{F})(500\ \text{V}) = 2.2 \times 10^{-8}\ \text{C} = 22\ \text{nC}$$

$$\text{Energy} = \tfrac{1}{2}qV = \tfrac{1}{2}(2.2 \times 10^{-8}\ \text{C})(500\ \text{V}) = 5.5 \times 10^{-6}\ \text{J} = 5.5\ \mu\text{J}$$

$$E = \frac{V}{d} = \frac{500\ \text{V}}{4.0 \times 10^{-3}\ \text{m}} = 1.3 \times 10^{5}\ \text{V/m}$$

(*c*) The capacitor will now have a capacitance $K = 2.60$ times larger than before. Therefore,

$$q = CV = (2.60 \times 4.4 \times 10^{-11}\ \text{F})(500\ \text{V}) = 5.7 \times 10^{-8}\ \text{C} = 57\ \text{nC}$$

The capacitor already had a charge of 22 nC and so 57 nC – 22 nC or 35 nC must have been added to it.

Supplementary Problems

Two metal plates are attached to the two terminals of a 1.50-V battery. How much work is required to carry a +5.0-μC charge (*a*) from the negative to the positive plate, (*b*) from the positive to the negative plate? *Ans.* (*a*) 7.5 μJ, (*b*) −7.5 μJ

The potential difference between two large parallel metal plates is 120 V. The plate separation is 3.0 mm. Find the electric field between the plates. *Ans.* 40 kV/m toward negative plate

Determine the absolute potential in air at a distance of 3.0 cm from a point charge of 500 μC. *Ans.* 15 kV

A capacitor with air between its plates has capacitance 3.0 μF. What is its capacitance when wax of dielectric constant 2.8 is placed between the plates? *Ans.* 8.4 μF

Determine the charge on each plate of a 0.050-μF capacitor when the potential difference between the plates is 200 V. *Ans.* 10 μC

A capacitor is charged with 9.6 nC and has a 120 V potential difference between its terminals. Compute its capacitance and the energy stored in it. *Ans.* 80 pF, 0.58 μJ

Compute the energy stored in a 60-pF capacitor (*a*) when it is charged to a potential difference of 2.0 kV and (*b*) when the charge on each plate is 30 nC. *Ans.* (*a*) 12 mJ; (*b*) 7.5 μJ

Three capacitors, each of capacitance 120 pF, are each charged to 0.50 kV and then connected in series. Determine (*a*) the potential difference between the end plates, (*b*) the charge on each capacitor, and (*c*) the energy stored in the system. *Ans.* (*a*) 1.5 kV; (*b*) 60 nC; (*c*) 45 μJ

Three capacitors (2.00 μF, 5.00 μF, and 7.00 μF) are connected in series. What is their equivalent capacitance? *Ans.* 1.19 μF

Three capacitors (2.00 μF, 5.00 μF, and 7.00 μF) are connected in parallel. What is their equivalent capacitance? *Ans.* 14.00 μF