Engineering Mechanics: Dynamics

الميكانيكا الهندسية: الديناميكا

Lec (3)

Curvilinear Motion: Rectangular Components

الحركة المنحنية: المركّبات المستطيلة (الإحداثيات الديكارتية)

Occasionally, the motion of a particle can best be described along a path that can be expressed in terms of its x, y, z coordinates.

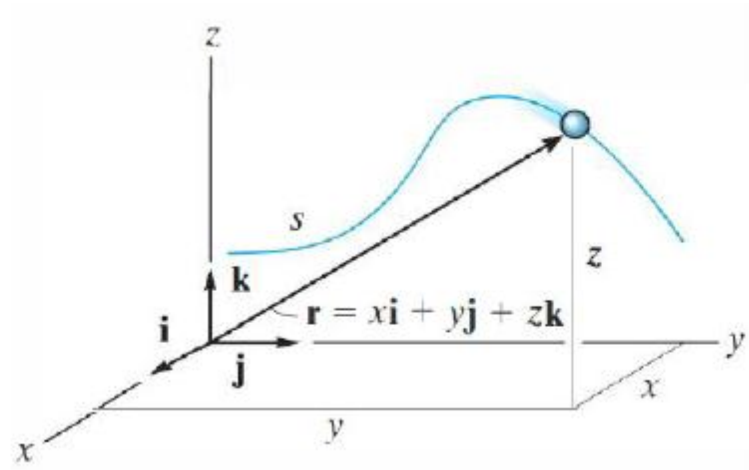
في بعض الأحيان، يمكن وصف حركة الجسيم بشكل أفضل على طول مسار يمكن التعبير عنه بدلالة إحداثياته x, y, z .

Position (الموقع)

If the particle is at point **(x, y, z)** on the curved path **s** , then its location is defined by the position vector:

إذا كان الجسيم عند النقطة **(x, y, z)** على المسار المنحني **s** ، فإن موقعه يُحدَّد بواسطة متجه الموقع (Position Vector):

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$



Position

(a)

When the particle moves, the **x, y, z** components of **r** will be functions of time; i.e., **x = x(t) , y = y(t) , z = z(t) ,** so that **r = r (t)** .

عندما يتحرك الجسيم، ستكون المركّبات **x, y, z** للمتجه **r** عبارة عن دلالات زمنية؛ أي أن **x = x(t)** و **y = y(t)** و **z = z(t)** ، بحيث يصبح المتجه **r = r (t)** .

At any instant, the magnitude of **r** is defined by:

في أي لحظة، يتم تحديد مقدار المتجه **r** بواسطة المعادلة:

$$r = \sqrt{x^2 + y^2 + z^2}$$

And the direction of **r** is specified by the unit vector:

واتجاه **r** يُحدَّد بواسطة متجه الوحدة:(Unit Vector)

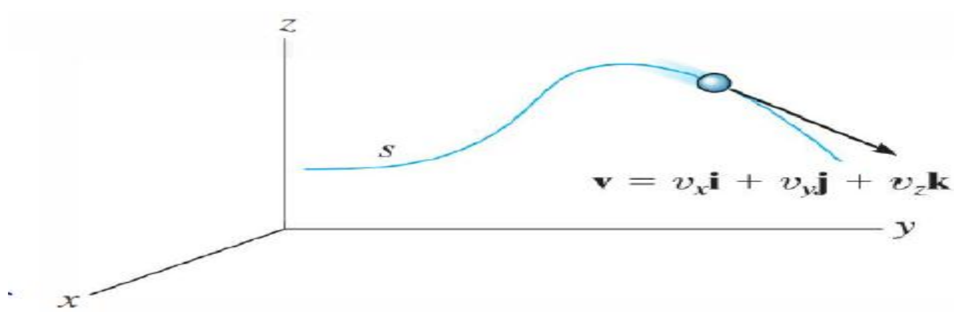
$$U_r = \vec{r} / r$$

Velocity (السرعة)

The first time derivative of **r** yields the velocity of the particle. Hence,

تُعطي المشتقة الزمنية الأولى للمتجه **r** سرعة الجسيم. وبناءً على ذلك:

$$V = dr/dt = dxi/dt + dyj/dt + dzk/dt$$



Velocity

(b)

When taking this derivative, it is necessary to account for changes in both the magnitude and direction of each of the vector's components. For example, the derivative of the **i** component of **r** is:

عند أخذ هذه المشتقة، من الضروري مراعاة التغيرات في كل من المقدار والاتجاه لكل مركّبة من مركّبات المتجه. على سبيل المثال، مشتقة المركّبة **i** للمتجه **r** هي:

$$v_x = dx/dt = \dot{x}$$

The second term on the right side is zero, provided the **x, y, z** reference frame is fixed, and therefore the direction (and the magnitude) of **i** does not change with time.

الحد الثاني على الجانب الأيمن يساوي صفرًا، بشرط أن يكون إطار الإسناد (النظام الإحداثي **x, y, z**) ثابتًا، وبالتالي فإن اتجاه (ومقدار) متجه الوحدة **i** لا يتغير مع الزمن.

Differentiation of the **j** and **k** components may be carried out in a similar manner, which yields the final result:

يمكن إجراء التفاضل لمركّبات **j** و **k** بطريقة مماثلة، مما يعطي النتيجة النهائية:

$$v_y = dy/dt = \dot{y} \qquad v_z = dz/dt = \dot{z}$$

The "dot" notation $\dot{x}, \dot{y}, \dot{z}$ represents the first time derivatives of $x = x(t)$, $y = y(t)$, $z = z(t)$, respectively.

تُمثّل نقطة التدوين (Dot notation) $\dot{x}, \dot{y}, \dot{z}$ المشتقات الزمنية الأولى لـ $x = x(t)$, $y = y(t)$, $z = z(t)$ على التوالي.

The velocity has a magnitude that is found from:

تمتلك السرعة مقدارًا يتم حسابه من :

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

And a direction that is specified by the unit vector $\mathbf{u}_v = \vec{\mathbf{v}}/v$. This direction is always tangent to the path.

واتجاهًا يُحدَّد بواسطة متجه الوحدة $\mathbf{u}_v = \vec{\mathbf{v}}/v$. ويكون هذا الاتجاه دائمًا مماسًا للمسار.

Acceleration (التسارع)

The acceleration of the particle is obtained by taking the first time derivative of the velocity (or the second time derivative of position):

يتم الحصول على تسارع الجسيم عن طريق أخذ المشتقة الزمنية الأولى للسرعة (أو المشتقة الزمنية الثانية للموقع):

$$\mathbf{a} = d\mathbf{v} / dt = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

Where:

$$a_x = \dot{v}_x = \ddot{x} = d^2x / dt^2$$

$$a_y = \dot{v}_y = \ddot{y} = d^2y / dt^2$$

$$a_z = \dot{v}_z = \ddot{z} = d^2z / dt^2$$

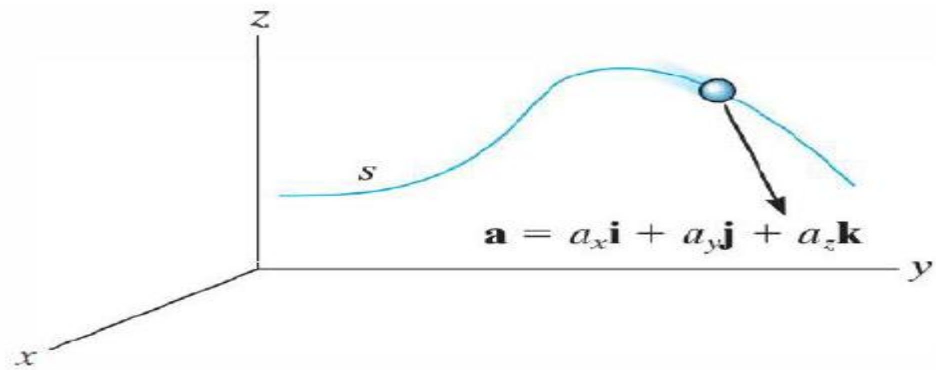
The acceleration magnitude is found from:

يتم حساب مقدار التسارع من خلال:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

And its direction is specified by the unit vector $\mathbf{u}_a = \vec{\mathbf{a}}/a$. Since **a** represents the time rate of change in both the magnitude and direction of the velocity, in general, **a** will not be tangent to the path.

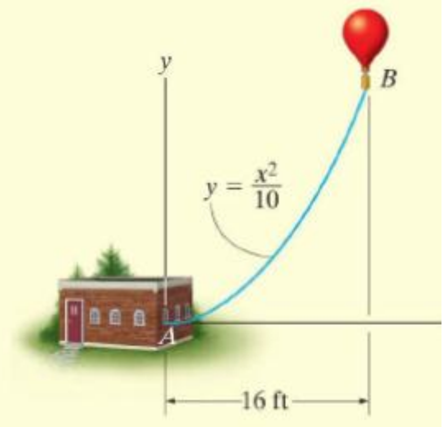
ويُحدَّد اتجاهه بواسطة متجه الوحدة $\mathbf{u}_a = \mathbf{a} / a$. وبما أن $\mathbf{a}$ يمثل معدل التغير الزمني في كل من مقدار السرعة واتجاهها، فإن المتجه $\mathbf{a}$ بصفة عامة لن يكون مماسًا للمسار.



Acceleration

(c)

At any instant the horizontal position of the weather balloon in Fig. 12–18a is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



(a)

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. (See Appendix A for a full explanation.)

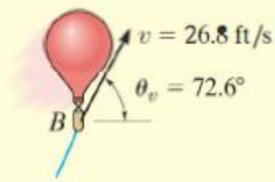
$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18b, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6^\circ \qquad \textit{Ans.}$$



(b)

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

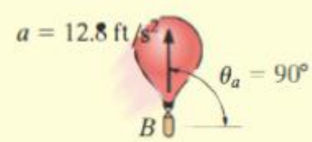
$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18c, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90^\circ \qquad \textit{Ans.}$$



(c)

Fig. 12–18

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

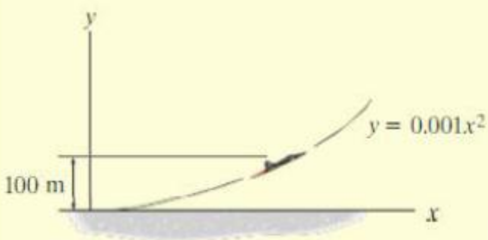
SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, since $v_y = 10$ m/s, then

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \tag{1}$$



(a)

Thus

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

The magnitude of the velocity is therefore

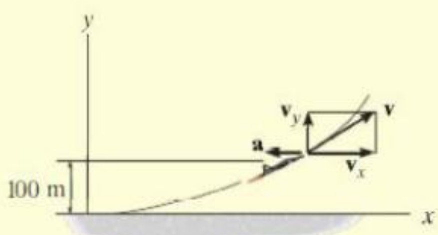
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \qquad \textit{Ans.}$$

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002((15.81\text{ m/s})^2 + 316.2\text{ m}(a_x))$$
$$a_x = -0.791\text{ m/s}^2$$



(b)

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2 \qquad \textit{Ans.}$$

Fig. 12–19