INTEGRATION التكـــــــــــــامل

SPECIAL METHODS OF INTEGRATION طرق خاصة للتكامل:

(1) INTEGRATION BY SUBISTITUTION التكامل بالتعويض:

اذا كان الدالة المعينة وتفاضلها موجودة داحل علامة التكامل ف من الممكن استبدال المتغيرات ومن المناسب اسنخدام الاتي: $u = g(x) \to du = g^{\backslash}(x)dx$

Many functions are formed by using composition in dealing with composite function if is useful to change variables of integrations it is convenient to use the following differentiation notation:

$u = g(x) \to du = g^{\backslash}(x)dx \to \int f(g(x))\, g^{\backslash}(x)dx = \int f(u)\, du$If

Example:

Evaluate the following integrals: احسب التكاملات الاتية:

(1) $\int xcosx^2dx$ (2) $\int 3x^2e^{x^2}\, dx$ (3) $\int sinx\, cosx\, dx$ (4) $\int \frac{dx}{(1+x^2)\tan^{-1}x}$

Solution:

(1) $\int xcosx^2dx$ $Let\, u = x^2 \to du = 2xdx$ $dx = \frac{du}{2x} \Rightarrow \int (xcosu)\,(\frac{du}{2x})$

$= \frac{1}{2}\int cosudu\, du = \frac{1}{2}sinx + c\frac{1}{2}sinx^2 + c$

(2) $\int 3x^2e^{x^3}\, dx$ Let $u = x^3 \Rightarrow du = 3x^2dx \Rightarrow dx = \frac{du}{3x^2} \Rightarrow$

$\int 3x^2e^{x^3}\, dx = \int 3x^2\left(e^u\frac{du}{3x^2}\right) = \int e^u\, du = e^u + c = e^{x^3} + c$

(3) $\int sinx\, cosx\, dx$ Let $u = sinx \Rightarrow du = cosxdx \Rightarrow dx = \frac{du}{cosx}$

$\int sinx\, cosx\, dx = \int (ucosu)\left(\frac{du}{cosu}\right) = \int u\, du = \frac{1}{2}u^2 = \frac{1}{2}sin^2x + x$

(4) $\int \frac{dx}{(1+x^2)\tan^{-1}x}$ Let $u = \tan^{-1}x \Rightarrow du = \frac{dx}{1+x^2} \Rightarrow dx = (1+x^2)du$

$\int \frac{dx}{(1+x^2)\tan^{-1}x} = \int \frac{(1+x^2)}{(1+x^2)u}du = \int \frac{du}{u} = \ln u + c = \ln(\tan^{-1}x) + c$

Example:

Evaluate the integrals احسب التكاملات الاتية:

(1) $\int_0^{\frac{\pi}{6}} tan2x\,dx$ (2) $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}}$

Solution:

(1) $\int_0^{\frac{\pi}{6}} tan2x\,dx$ Let $u = 2x \Rightarrow du = 2dx \Rightarrow dx = \frac{du}{2} \Rightarrow$

$\int_0^{\frac{\pi}{6}} tan2x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} tan\,u\,du = \frac{1}{2}[lnsecu]_0^{\frac{\pi}{3}} = \frac{1}{2}[ln2 - ln1] = \frac{1}{2}ln2$

(2) $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} = \int_1^2 \frac{dx}{((x-1)^2+3)^{\frac{3}{2}}}$ Let $x-1 = \sqrt{3}tanu \Rightarrow dx = \sqrt{3}secudu$

$\therefore \int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} = \int_1^{\frac{\pi}{6}} \frac{\sqrt{3}secudu}{\left((\sqrt{3}\,tanu)^2+3\right)^{\frac{3}{2}}} = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}secudu}{(3tan^2x+3)^{\frac{3}{2}}} = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}tanudu}{((x-1)^2+3)^{\frac{3}{2}}} = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}secudu}{(3sec^2u)^{\frac{3}{2}}}$

$= \frac{1}{3}\int_0^{\frac{\pi}{6}} cosudu\,du = \frac{1}{3}[sinu]_0^{\frac{\pi}{6}} = \frac{1}{6}$

Example:

Evaluate the integrals احسب التكاملات الاتية:

(1) $\int (x+2)sin(x^2+4x-6)\,dx$ (2) $\int \frac{dx}{\sqrt{(x+2)(3-x)}}$ (3) $\int \frac{x\,sin^{-1}x^2\,dx}{\sqrt{-x^4}}$

(4) $\int \frac{cot(lnx)}{x}dx$ (5) $\int 2^{1-x}tanh2^{1-x}\,dx$ (6) $\int \frac{xdx}{\sqrt{x^2+x+1}}$

Solution:

(1) $\int (x+2)sin(x^2+4x-6)\,dx$ $u = (x^2+4x-6) \Rightarrow$

$du = (x+x)dx$

$\int (x+2)sin(x^2+4x-6)\,dx = \frac{1}{2}\int sinu\,du = -\frac{1}{2}cosu + c = sin(x^2+4x-6) - \frac{1}{2} + c$

(2) $\int \frac{dx}{\sqrt{(x+2)(3-x)}} = \int \frac{dx}{\sqrt{6-x-x^2}} = \int \frac{dx}{\sqrt{\frac{25}{4}-(x-\frac{1}{2})^2}}$

$u = x - \frac{1}{2} \Rightarrow du = dx \Rightarrow$

$$\int \frac{dx}{\sqrt{\frac{25}{4} - (x-\frac{1}{2})^2}} = \int \frac{du}{\sqrt{\frac{25}{4} - u^2}} = \sin^{-1}\left(\frac{u}{\frac{5}{2}}\right) = \sin^{-1}\left(\frac{2x-1}{5}\right) + c$$

(3) $\int \frac{x \sin^{-1} x^2 \, dx}{\sqrt{-x^4}}$ let $u = \sin^{-1} x^2 \Rightarrow du = \frac{2x dx}{\sqrt{1-x^4}}$

$$\int \frac{x \sin^{-1} x^2 \, dx}{\sqrt{-x^4}} = \frac{1}{2}\int u \, du = \frac{1}{4}(\sin^{-1} x^2)^2 + c$$

(4) $\int \frac{cot(lnx)}{x} dx$ Let $u = lnx \Rightarrow ud = \frac{dx}{x}$

$$\therefore \int \frac{cot(lnx)}{x} dx = \int cotu \, du = lnsinu + c = \ln(sin(ln)) + c$$

(5) $\int 2^{1-x} tanh 2^{1-x} \, dx$ Let $u = 2^{1-x} \Rightarrow du = -2^{1-x}(ln2) \Rightarrow 2^{1-x} dx = -\frac{du}{ln2}$

$$\therefore \int 2^{1-x} tanh 2^{1-x} \, dx = -\frac{1}{ln2}\int tanh \, du = -\frac{1}{ln2} lncoshu + c = \frac{1}{ln2} lncosh 2^{1-x} + c$$

$$(6) \int \frac{xdx}{\sqrt{x^2+x+1}} = \frac{1}{2}\left(\int \frac{2x+1-1}{\sqrt{x^2+x+1}} dx\right) = \frac{1}{2}\left(\int \frac{2x+1}{\sqrt{x^2+x+1}} - \int \frac{1}{\sqrt{x^2+x+1}}\right) dx =$$

$$= \sqrt{x^2+x+1} - \frac{1}{2}\int \frac{dx}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1} - \frac{1}{2}\int \frac{dx}{\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \sqrt{x^2+x+1} - \frac{1}{2}\left(ln\left|\left(x+\frac{1}{2}\right)\right| + \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\right) + c$$

تكامل النسب المثلثية التربيعية

Example:

Evaluate the integrals احسب التكاملات الاتية:

(1) $\int sin^2 x \, dx$ (2) $\int cos^2 x \, dx$ (3) $\int tan^2 x \, dx$

(4) $\int cot^2 x \, dx$ (5) $\int sec^2 x \, dx$ (6) $\int csc^2 x \, dx$

Solution:

(1) $\int sin^2x\,dx = \frac{1}{2}\int (1-cos2x)\,dx = \frac{1}{2}\left[x - \frac{cos2x}{2}\right] + c$

(2) $\int cos^2 x\,dx = \frac{1}{2}\int (1+cos2x)\,dx = \frac{1}{2}\left[x + \frac{cos2x}{2}\right] + c$

(3) $\int tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = tanx - x + c$

(4) $\int cot^2 x\,dx = \int (\csc^2 x - 1)\,dx = -cotx - x + c$

(5) $\int sec^2 x\,dx = tanx + c$

(6) $\int csc^2 x\,dx = -cotx + c$

Example:

Evaluate the integrals احسب التكاملات الاتية:

(1) $\int sinh^2x\,dx$ (2) $cosh^2x\,dx$ (3) $\int tanh^2x\,dx$ (4) $\int coth^2x\,dx$

Solutions:

We have $cosh^2x - sinh^2x = 1$ and $sech^2x + tanh^2x = 1$ and

$sinh^2x = \frac{1}{2}(cosh2x - 1)$ and $cosh^2x = \frac{1}{2}(1 + cosh2x)$

(1) $\int sinh^2x\,dx = \int \frac{1}{2}(cosh2x - 1)\,dx = \frac{1}{2}\left[\frac{sinh2x}{2} - x\right] + c$

(2) $cosh^2x\,dx = \int \frac{1}{2}(cosh2x + 1)\,dx = \frac{1}{2}\left[\frac{sinh2x}{2} + x\right] + c$

(3) $\int tanh^2x\,dx = \int (1 - sech^2x)\,dx = x - tanhx + c$

(4) $\int coth^2x\,dx = \int (1 + sech^2x)\,dx = x - cothx + c$

Exercise:(7)

(1) $\int e^{x^3}x^2\,dx$ (2) $\int x^3 sinh\,(x^2+6)\,dx$ (3) $\int \frac{(sin^{-1}x)^4}{\sqrt{1-x^2}}dx$ (4) $\int x\sqrt{x^2-3}\,dx$

(5) $\int \frac{(arctanx)^3}{1+x^2}dx$ (6) $\int x^3 lnx\,dx$ (7) $\int \frac{x^2}{1+x^6}dx$ (8) $\int 2^{\sqrt{2x+1}}\,dx$

(9) $\int e^x sine^x\,dx$ (10) $\int csc\,(5c-3)\,dx$ (11) $\int xtan(x^2-1)\,dx$

(2) INTEGRATION BY PARTS: التكامل التجزئة:

د.هشام عبدالله عثمان