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INTEGRATION التكــــــــــــــامل

بعض التكاملات

Example:

Evaluate the integrals احسب التكاملات الاتية:

(1) $\int sin^2 x\, dx$ (2) $\int cos^2 x\, dx$ (3) $\int tan^2 x\, dx$

(4) $\int cot^2 x\, dx$ (5) $\int sec^2 x\, dx$ (6) $\int csc^2 x\, dx$

Solution:

(1) $\int sin^2 x\, dx = \frac{1}{2}\int (1 - cos2x)\, dx = \frac{1}{2}\left[x - \frac{cos2x}{2}\right] + c$

(2) $\int cos^2 x\, dx = \frac{1}{2}\int (1 + cos2x)\, dx = \frac{1}{2}\left[x + \frac{cos2x}{2}\right] + c$

(3) $\int tan^2 x\, dx = \int (\sec^2 x - 1)\, dx = tanx - x + c$

(4) $\int cot^2 x\, dx = \int (\csc^2 x - 1)\, dx = - cotx - x + c$

(5) $\int sec^2 x\, dx = tanx + c$

(6) $\int csc^2 x\, dx = - cotx + c$

Example:

Evaluate the integrals احسب التكاملات الاتية:

(1) $\int sinh^2 x\, dx$ (2) $cosh^2 x\, dx$ (3) $\int tanh^2 x\, dx$ (4) $\int coth^2 x\, dx$

Solutions:

We have $cosh^2 x - sinh^2 x = 1$ and $sech^2 x + tanh^2 x = 1$ and

$sinh^2 x = \frac{1}{2}(cosh2x - 1)$ and $cosh^2 x = \frac{1}{2}(1 + cosh2x)$

(1) $\int sinh^2 x\, dx = \int \frac{1}{2}(cosh2x - 1)\, dx = \frac{1}{2}\left[\frac{sinh2x}{2} - x\right] + c$

(2) $cosh^2 x\, dx = \int \frac{1}{2}(cosh2x + 1)\, dx = \frac{1}{2}\left[\frac{sinh2x}{2} + x\right] + c$

$(3)\ \int tanh^2x\,dx = \int (1-\ sech^2x)\,dx\ = x - tanhx + c$

$(4)\ \int coth^2x\,dx = \int (1+\ sech^2x)\,dx\ = x - cothx + c$

Exercise:(7)

$(1)\ \int e^{x^3}x^2\,dx\quad (2)\int x^3 sinh\,(x^2+6)\,dx\quad (3)\ \int \frac{(sin^{-1}x)^4}{\sqrt{1-x^2}}dx\quad (4)\ \int x\sqrt{x^2-3}\,dx$

$(5)\ \int \frac{(arctanx)^3}{1+x^2}dx\quad (6)\ \int x^3 lnx\,dx\quad (7)\ \int \frac{x^2}{1+x^6}dx\quad (8)\ \int 2^{\sqrt{2x+1}}\,dx$

$(9)\ \int e^x sine^x\,dx\quad (10)\ \int csc\,(5c-3)\,dx\quad (11)\ \int xtan(x^2-1)\,dx$

(2) INTEGRATION BY PARTS: **التكامل التجزئة:**

اذا كانت دوال u و v قابلة للتفاضل اعتمادا على قانون تفاضل ضرب دالتين $d(uv) = udv + vdu$ وباخذ التكامل لطرفي المعادلة نجد ان: $\int udv = uv - \int v\,du$ وصيغة التكامل بالتجزئة تكتب بالصورة الاتية:

$\int u\,dv = uv - \int v\,du$

$d(vv) =$ be differential functions. According to the product rule for differentials v and uLet $udv + vdu$

Upon taking the antidervatives of both sides of the equation, we obtain

$uv = \int u\,dv + \int v\,du$

$\int u\,dv = uv - \int v\,du$ This is the formula of integration by parts when in the form

then $[a,b]$ be continuous on an interval $f(x), g(x), f^{\backslash}(x), g^{\backslash}(x)$Or if

او اذا كانت $f(x), g(x), f^{\backslash}(x), g^{\backslash}(x)$ دوال مستمرة في الفترة $[a,b]$ عليه نجد ان:

$(i)\ \int f(x)g^{\backslash}(x)\,dx = f(x)g(x) - \int g(x)\,f^{\backslash}(x)dx$

$(ii)\ \int_a^b \int f(x)g^{\backslash}(x)\,dx\ = [f(x)g(x)]_a^b - \int_a^b \int g(x)\,f^{\backslash}(x)dx$

Where $u = f(x), dv = g^{\backslash}(x)$ are parts of the integration?

بحيث ان $u = f(x), dv = g^{\backslash}(x)$ تمثل اجزاء من التكامل.

Example:

Evaluate the following integrals احسب التكاملات الاتية:

(1) $\int x\, sinx\, dx$ (2) $\int x\, e^{4x}dx$ (3) $\int x^2\, cosxdx$ (4) $\int lnx\, dx$ (5) $\int x^3 e^{x^2}\, dx$

Solution

(1) $\int x\, sinx\, dx$

Let $u = x \Rightarrow du = dx$ and $dv = sinxdx$ $v =- cosx$

$\therefore \int x\, sinx\, dx =- xcosx - \int -cosx\, dx =- xcosx + sinx + c$

(2) $\int x\, e^{4x}dx$

Let $u = x \Rightarrow du = dx$ and $dv = e^{4x}dx \Rightarrow \int dv = \int e^{4x}\, dx \Rightarrow v = \frac{1}{4}e^{4x}$

$\therefore \int x\, e^{4x}dx = \frac{x}{4}e^{4x} - \int \frac{1}{4}e^{4x}\, dx = \frac{x}{4}e^{4x} - \frac{1}{16}e^{4x} + c$

(3) $\int x^2\, cosxdx$

Let $u = x^2 \Rightarrow du = 2xdx$ and $dv = cosxdx \Rightarrow v = sinx$

$\therefore \int x^2\, cosxdx = x^2 sinx - \int 2xsinx\, dx = 2\int x\, sinx\, dx$

Again let $u = x \Rightarrow du = dx$ and $dv = sinxdx \Rightarrow v =- cosx$

$\Rightarrow 2\int x\, sinx\, dx =- 2xcosx - 2\int -cosx\, dx = 2[-xcosx + sinx]$

$\therefore \int x^2\, cosxdx \quad = x^2 sinx - 2[-xcosx + sinx] + c$

(4) $\int lnx\, dx$

Let $u = lnx \Rightarrow du = \frac{dx}{x}$ and $dv = dx \Rightarrow v = x$

$\therefore \int lnx\, dx = xlnx - \int x\frac{dx}{x} = xlnx - \int dx = xlnx - x + c$

(5) $\int x^3 e^{x^2}\, dx$

Let $u = x^2 \Rightarrow du = 2xdx$ and $v = xe^{x^2} \Rightarrow dv = \frac{1}{2}e^{x^2}$

$\therefore \int x^3 e^{x^2}\, dx = \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + c$

Example:

Evaluate the following integrals احسب التكاملات الاتية:

(1) $\int cos^{-1} x\, dx$ (2) $\int tan^{-1} x\, dx$ (3) $\int x\, sin^{-1} x\, dx$ (4) $\int x\, sec^{-1} x\, dx$

Solution:

(1) $\int cos^{-1} x\, dx$

Let $u = \cos^{-1} x \Rightarrow x = cosu \Rightarrow dx =- sinudu$

$\therefore \int cos^{-1} x\, dx = x \cos^{-1} x - \sin(\cos^{-1} x) + c$

(2) $\int tan^{-1} x\, dx$

Let $u = \tan^{-1} x \Rightarrow x = tanu \Rightarrow dx = \int sec^2udu$

$\therefore \int tan^{-1} x\, dx = \int usec^2udu = \tan^{-1} x \tan(\tan^{-1} x) + \ln\cos(\tan^{-1} x) + \mathrm{c}$

(3) $\int x\, sin^{-1} x\, dx$

Let $u = \sin^{-1} x \Rightarrow x = sinu \Rightarrow dx = cosudu$

$\int x\, sin^{-1} x\, dx = \int u\, sinu\, cosudu = \frac{1}{2}\int usin2udu =$

$\frac{1}{2}\left[\frac{-1}{2}ucos2u + \frac{1}{4}sin2u\right] = \frac{1}{4}\left(\sin^{-1} x\, cos(2\sin^{-1} x)\right) + sin(2\sin^{-1} x) + c$

(4) $\int x\, sec^{-1} x\, dx$

Let $u = sec^{-1} x \Rightarrow x = secu \Rightarrow dx = secu\, tanudu$

$\int x\, sec^{-1} x\, dx = \int u\, secu\,(secu\, tanu)du = \int u\, sec^2u\, tanudu$

$= \frac{1}{1}utan^2u - \frac{1}{2}(u - tanu) = \frac{1}{2}\sec^{-1} x\, tan^2(\sec^{-1} x) - \frac{1}{2}\left(\sec^{-1} x - tan(\sec^{-1} x)\right) + c$

Example:

Evaluate the following integrals احسب التكاملات الاتية:

(1) $\int ln(x^2 + 2)\, dx$ (2) $\int sec^3x\, dx$

Solution:

(1) $\int ln(x^2+2)\,dx$ Let $u = ln(x^2+2) \Rightarrow du = \frac{2xdx}{x^2+2}$ and $dv = dx \Rightarrow v = x$

$\int ln(x^2+2)\,dx = xln(x^2+2) - 2\int \frac{x^2dx}{x^2+2} = xln(x^2+2) - 2\int \left(1 - \frac{2}{x^2+2}\right) dx$

$= xln(x^2+2) - 2x + \frac{4}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

(2) $\int sec^3 x\,dx$

Let $u = \sec x \Rightarrow du = escx\,tanxdx$ and $dv = sec^2 x \Rightarrow v = tanx$

$I = \int sec^3 x\,dx = \int secx\,sec^2 x\,dx = secx\,tanx - \int secx\,tan^2 x\,dx \Rightarrow$

$\int secx\,tan^2 x\,dx = \int secx(sec^2 x - 1)\,dx = \int sec^3 x\,dx - \int secx\,dx =$

$I - ln|secx + tanx|$

$\therefore I = \int sec^3 x\,dx = secx\,tanx - [I - ln|secx + tanx|] =$

$secx\,tanx - I + ln|secx + tanx|$

$\therefore I = \int sec^3 x\,dx = secx\,tanx - I + ln|secx + tanx| \Rightarrow$

$I + I = 2I = secx\,tanx + ln|secx + tanx|$

$\therefore I = \int sec^3 x\,dx = \frac{1}{2}(secx\,tanx + ln|secx + tanx|)$ +c

Exercise:8

(1) $\int x^2 \cos x^3\,dx$ (2) $\int csc^{-1} x\,dx$ (3) $\int sec2x\,tan2x\,dx$ (4) $\int \frac{1}{x^2} cos^2\left(\frac{1}{x}\right) dx$

(5) $\int \sqrt{1 - cos2x}\,dx$ (6) $\int \frac{sexcxtanx}{\sqrt{secx}}\,dx$ (7) $\int xe^{x^2}\,dx$ (8) $\int sin2c\,sin4x\,dx$

(9) $\int \frac{1}{4+5sin2x}\,dx$ (10) $\int sin^{-1} 3x\,dx$ (11) $\int xcos^2 x\,dx$ (12) $\int e^x sinx\,dx$

(13) $\int (x^2 - 3x)e^x\,dx$ (14) $\int xsinh^2(x^2)\,dx$ (15) $\int x\sqrt{x^2 - 3}\,dx$ (16) $\int cot^7 x\,dx$

Prove that:

(1) $\int cotx\, dx = lnsinx + c$ (2) $\int \frac{sec^2x}{1-4tan^2x} dx = \frac{1}{2} sin^{-1}\,(2tanx)$

(3) $\int secx\, dx = ln|secx + tanx|$

(3) INTEGRATION BY PARTIAL FRACTIONS: التكامل بالكسور الجزئية: