INTEGRATION التكـــــــــــــامل

(2) EXPONENTIAL AND LOGARITHMIC FUNCTIONS الدوال الاسية واللوغرثمية:

With the fundamental theorems of calculus it is possible to rigorously develop the logarithmic exponential

(3) $(i)\int a^{bx}\,dx=\frac{a^{bx}}{blna}+c \quad (a>0\,,\ a\neq 1)$

$(ii)\ \int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$

(4) $\int ln|x|\,dx=xlnx-x+c$

Examples:

Evaluate the following integrals: (احسب التكاملات الاتية)

(1) $\int 3^{5x}dx$ (2) $\int\left(\frac{1}{7}\right)^{5x}dx$ (3) $\int e^{8x}\,dx$ (4) $\int\frac{\log_3 x}{x}dx$ (5) $\int xe^{x^2}\,dx$

Solution:

(1) $\int 3^{5x}dx=\frac{e^{5x}}{5ln3}+c$

(2) $\int\left(\frac{1}{7}\right)^{5x}dx=\int(7)^{-9x}\,dx=\frac{7^{-9x}}{-9ln7}+c$

(3) $\int e^{8x}\,dx=\frac{1}{8}e^{8x}+c$

(4) $\int\frac{\log_3 x}{x}dx=\frac{1}{ln3}\int\frac{lnx}{x}dx=\frac{(lnx)^2}{2ln3}+c$ where $\log_3 x=\frac{lnx}{ln3}$

(5) $\int xe^{x^2}\,dx=\frac{1}{2}\int xe^{x^2}\,d(x^2)=\frac{1}{2}e^{x^2}+c$

Exercise (1):

Evaluate the following integrals: (احسب التكاملات الاتية)

(1) $\int 2^{-x}\,dx$ (2) $\int 3x^{\sqrt{3}}dx$ (3) $\int x^{\sqrt{2}-1}\,dx$ (4) $\int\int_1^{\sqrt{2}}x(2)^{x^2}\,dx$

(5) $\int_0^{\frac{\pi}{2}}7^{cost}\,dt$ (6) $\int_0^{\frac{\pi}{4}}\left(\frac{1}{3}\right)^{tanx}dx$ (7) $\int\left(\sqrt{2}+1\right)x^{\sqrt{2}}\,dx$ (8) $\int_1^e x^{(ln2)-1}\,dx$

(9) $\int_1^2 \frac{2^{lnx}}{x} dx$ (10) $\int_0^3 3(1+lnx)\,dx$ (11) $\int_1^4 \frac{2\sqrt{x}}{\sqrt{x}} dx$ (12) $\int \frac{log_{10} x}{x} dx$

(13) $\int_1^e \frac{2ln10\, log_{10} x}{x} dx$ (14) $\int_0^2 \frac{log_2 (x+2)}{(x+2)} dx$ (15) $\int \frac{1}{x(log_7 x)^2} dx$ (16) $\int \frac{dx}{x\, log_{10} x} dx$

(17) $\int_1^{ln2} \frac{1}{t} dt$ (18) $\int x^5 e^{x^6}\, dx$ (19) $\int x(x^2+3)^{-4}\, dx$

(3) Trigonometric functions **الدوال المثلية:**

(1) $\int sin\, x dx = -cosx + c$ (2) $\int cos\, dx = -sinx + c$

(3) $\int tanx\, dx = ln|secx| = -ln|cosx| + c$ (4) $\int cotx\, dx = |sinx| + c$

(5) $\int secx\, dx = ln|secx + tanx| + c$ (6) $\int cscx\, dx = ln|cscx - cot| + c$

(7) $\int sec^2 x\, dx = tanx + c$ (8) $\int csc^2 x\, dx = -cotx + c$

(9) $\int secxtanx\, dx = secx + c$ (10) $\int cscx\, cotx dx = -cscx + c$

Examples:

Evaluate the following integrals: (احسب التكاملات الاتية)

(1) $\int sin2x\, dx$ (2) $\int_0^{\frac{\pi}{4}} cos2\pi x\, dx$ (3) $\int tanx\, dx$ (4) $\int cot3x\, dx$

Solutions:

(1) $\int sin2x\, dx = \frac{-1}{2} cos2x + c$

(2) $\int_0^{\frac{\pi}{4}} cos2\pi x\, dx = \frac{1}{2\pi}[sin2\pi x]_0^{\frac{\pi}{4}} + c = \frac{1}{2\pi}\left(sin\frac{\pi}{2} - sin0\right) = \frac{1}{2\pi} + c$

(3) $\int tanx\, dx = -lncosx + x$ (4) $\int cot3x\, dx = \frac{1}{3} lnsin3x + c$

Example:

Evaluate the following integrals: (احسب التكاملات الاتية)

(1) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{4cosx}{3+2sinx} dx$ (2) $\int (tan2x + secxtanx)\, dx$

Solution:

$$(1)\ \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{4cosx}{3+2sinx}\,dx = 2[ln|3+2sinx|]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = 2ln\left|\frac{-\pi}{2}\right| - 2ln\left|\frac{\pi}{2}\right| = 2ln|1| - 2ln|5| = -2ln5$$

$$(2)\ \int (tan2x + secxtanx)\,dx = \int tan2x\,dx + \int (secxtanx)\,dx = \frac{1}{2}\big(\ln(sec2x)\big) + secx + c$$

Examples:

Calculate the following integrals: (احسب التكاملات الاتية)

(1) $\int sec2x\,dx$ (2) $\int csc^2 7x\,dx$ (3) $\int csc9x\,dx$

Solution:

(1) $\int sec2x\,dx = \frac{1}{2}ln|sec2x + tan2x| + c$

(2) $\int csc^2 7x\,dx = \frac{-1}{7}cot7x + c$

(3) $\int csc9x\,dx = \frac{1}{9}ln|csc9x - cot9x| + c$

Example:

Prove the following integrals: (احسب التكاملات الاتية)

(1) $\int tanx\,dx = -lncosx$ (2) $\int secx\,dx = ln|secx + tanx|$

Solution:

$$(1)\ \int tanx\,dx = \int \frac{sinx}{cosx}dx = -\int \frac{-sinx}{cosx}dx = -lncosx$$

$$(2)\ \int secx\,dx = \int \frac{secx(secx+tanx)}{secx+tanx}dx = ln|secx + tanx|$$

Exercise (2):

(A) Find the simplest ant derivatives you can for each part and add the arbitrary constant of integration at the end.

احسب ابسط صورة ممكنة للتكاملات الاتية ثم اضف ثابت التكامل لكل حالة.

(1) $\int (3x^2 + secx)\,dx$ (2) $\int \left(2x - cos7x + \sqrt{2x}\right)dx$ (3) $\int (tan6x + sec^2 x)\,dx$

(4) $\int (5^{3x} + e^{-x} + x^{-1})\,dx$ (5) $\int (4-8x)^9\,dx$ (6) $\int \left(\frac{t\sqrt{t}+\sqrt{t}}{t^2}\right)dt$

(7) $\int \frac{sec^2x}{4} dx$ (8) $\int 5secxtanx\, dx$ (9) $\int 6cos3x\, dx$

(10) $\int \left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right) dx$ (11) $\int x^{-3}(x+1)\, dx$ (12) $\int \frac{1}{2}\sqrt[5]{x}\, dx$

(13) $\int (x-4)(x+2)\, dx$ (14) $\int sec^2xtanx\, dx$ (15) $\int x^{-1}lnx\, dx$

(16) $\int xsinh(x^2)\, dx$ (17) $\int x\, tanh(x^2+1)\, dx$ (18) $\int \sqrt{1-cos^2x}\, dx$

(19) $\int tan^2x\, dx$ (20) $\int sin^3xcosx\, dx$ (21) $\int \frac{e^x-1}{e^x} dx$

(22) $\int xe^{x^2}\, dx$ (23) $\int e^xcosx\, dx$ (24) $\int cosx\, sinx\, dx$

(25) $\int \frac{x^2}{x^2+4} dx$ (26) $\int sin^3xcos^5x\, dx$ (27) $\int \frac{(sinx)^4}{\sqrt{1-cos^2x}} dx$

(28) $\int e^xsin\,(e^x)\, dx$ (29) $\int \frac{1}{x-4} dx$ (30) $\int lnx\, dx$

(B) **Prove that:**

(1) $\int cotx\, dx = lnsinx$ (2) $\int tanx\, dx = ln|secx|$

(3) $\int cscx\, dx = ln|cscx - cotx|$

(4) HYPERBOLIC FUNCTIONS: الدوال الزائدية

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