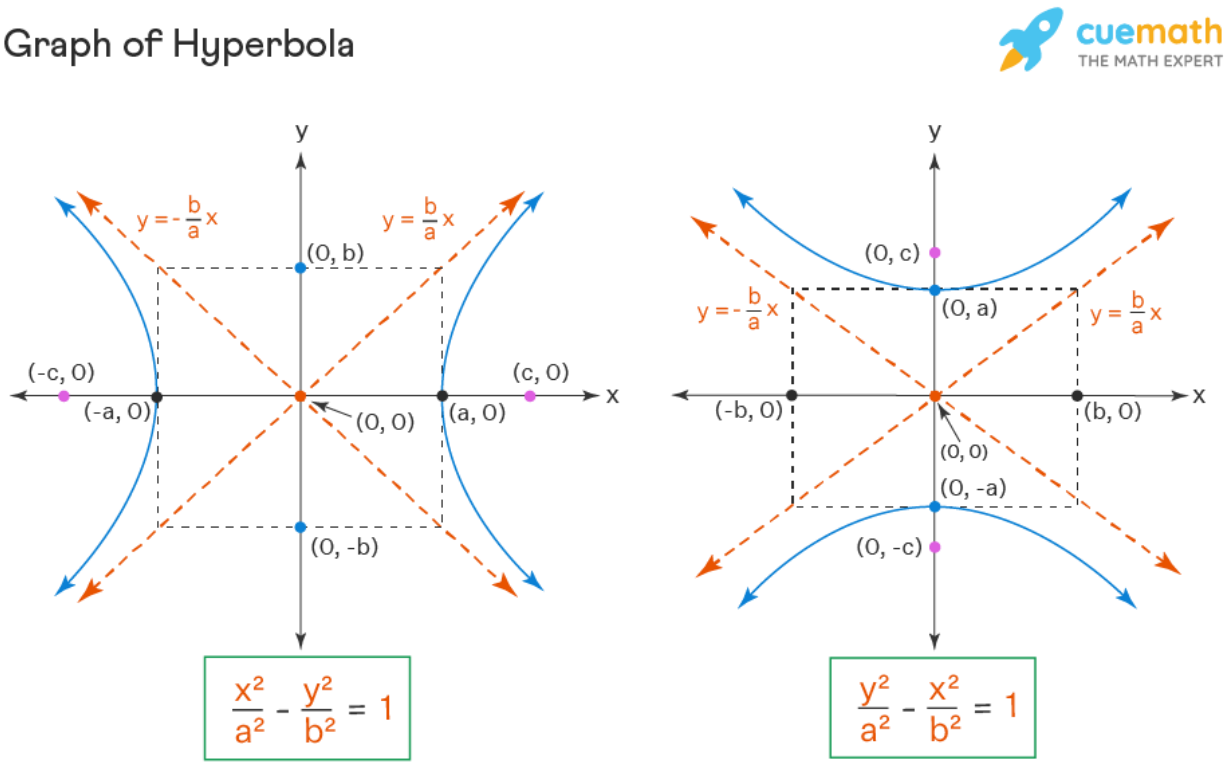
Lec(9)

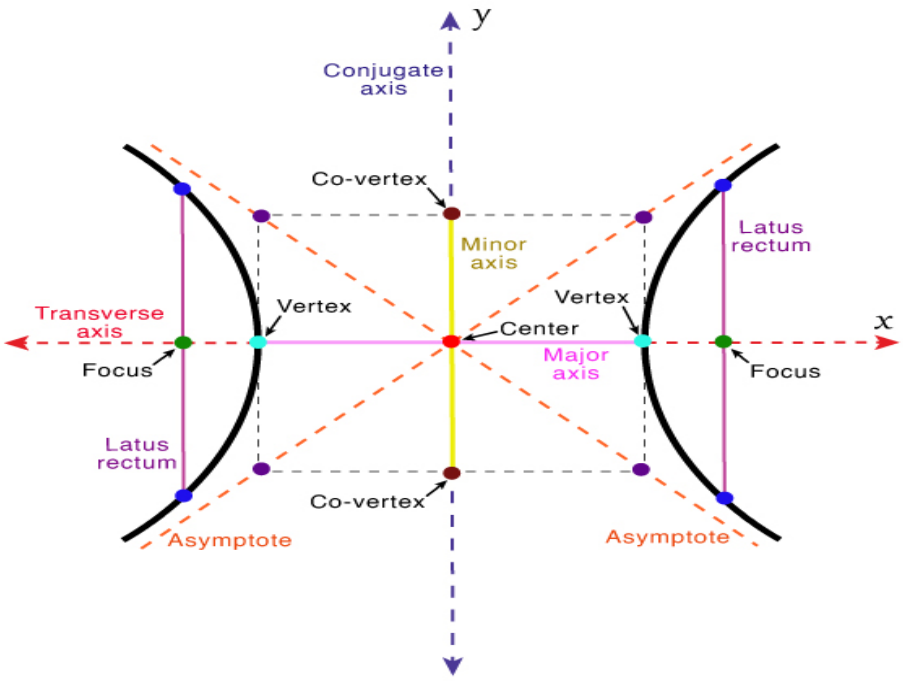
The hyperbola with center (0, 0)

Definition:-



The hyperbola is the of all point in a plane, the difference of whose distances from two fixed points is a given positive constant 2a.

Parts of a Hyperbola

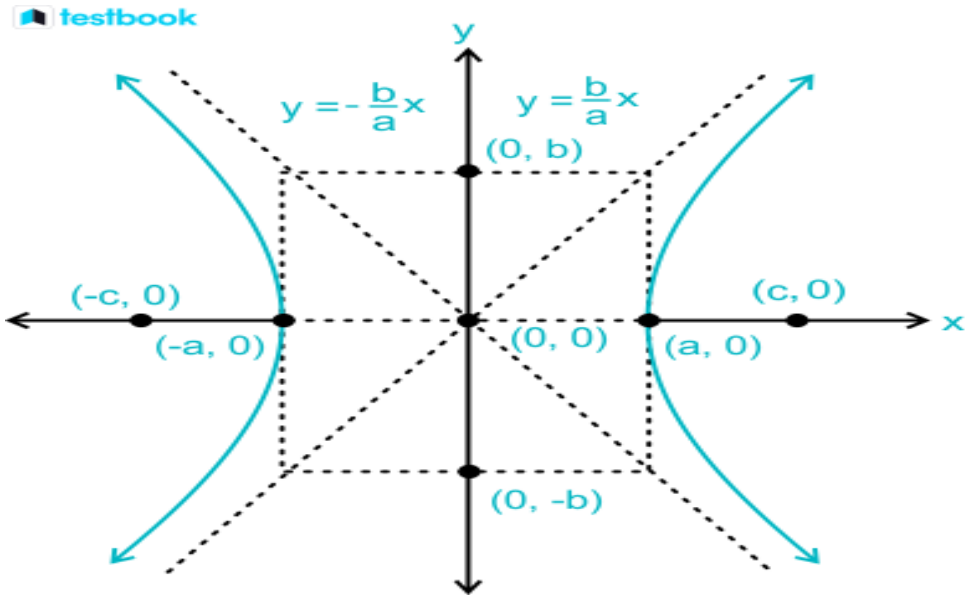


The distance between the foci equal 2c.

The distance between the vertices" ends of the focal axis "equal 2a.

The distance between the conjugate axis equal 2b.

case(1):-



The equation of the hyperbola with center (0,0) , the focal axis on the x-axis , vertices (a,0),(-a,0) , foci (c,0),(-c,0) and ends of the conjugate axis (0,b),(0,-b) is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (c^2 = a^2 + b^2)$$

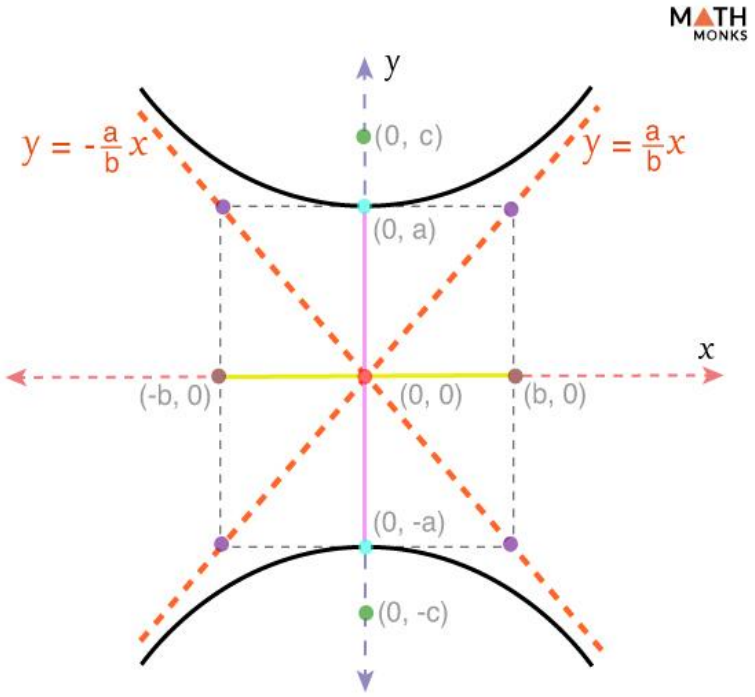
Asymptotes lines:-

The two lines through the center are called asymptotes lines

(The focal axis on the x-axis)

$$y = \pm \frac{b}{a} x$$

Case(2):-



$$y = \pm \frac{a}{b}x$$

here,

a = distance from the center to a vertex

b = distance from the center to a co-vertex

The equation of the ellipse with center (0,0) , the major axis on the y-axis , vertices (0,a),(0,-a) , foci (0,c),(0,-c) ends of the conjugate axis (b,0),(-b,0) is

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 \qquad (c^2=a^2+b^2)$$

Asymptotes lines:-

The two lines through the center are called asymptotes lines

(the focal axis on the y-axis)

$$y=\pm\frac{a}{b}x$$

Example:-

Find the equation of the hyperbola if

1) **center (0,0) , the foci $(\pm 6, 0)$,and vertices $(\pm 4, 0)$**

Solution:-

$$c=\pm 6$$
$$a=\pm 4$$
$$c^2=a^2+b^2$$
$$36=16+b^2 \Rightarrow b^2=36-16 \Rightarrow b^2=20$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{16}-\frac{y^2}{20}=1$$

2) center (0,0) , the foci $(0,\pm 10)$,and ends of focal axis $(0,\pm 6)$

Solution:-

$$c=\pm 10 \quad , \quad a=\pm 6$$

$$c^2=a^2+b^2$$

$$100=36+b^2 \Rightarrow b^2=100-36 \Rightarrow b^2=64$$

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

$$\frac{y^2}{36}-\frac{x^2}{64}=1$$

3) center (0,0) , ends of the conjugate axis $(0,\pm 3)$,and the length of the focal axis equal 8 units

Solution:-

$$b=\pm 3$$

$$2a=8 \Rightarrow a=4$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

4) center (0,0) , vertices $(\pm 2, 0)$, and asymptotes lines $y = \pm 3x$

Solution:-

$$a = \pm 2$$

$$y = \pm 3 = \pm \frac{b}{a} x \Rightarrow \frac{b}{a} = 3 \Rightarrow \frac{b}{2} = 3 \Rightarrow b = 6$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4} - \frac{y^2}{36} = 1$$

Example (2):-

Find the center, vertices, foci, ends of the conjugate axis, and asymptotes of the following equations

1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Solution:-

$$a^2 = 9 \Rightarrow a = \pm 3$$

$$b^2 = 4 \Rightarrow b = \pm 2$$

$$c^2 = a^2 + b^2$$

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 9 + 4 \Rightarrow c^2 = 13 \Rightarrow c = \pm\sqrt{13}$$

The center (0,0)

The vertices $(\pm a, 0) = (\pm 3, 0)$

The foci $(\pm c, 0)) = (\pm\sqrt{13}, 0)$

Ends of the conjugate axis $(0, \pm b) = (0, \pm 2)$

Asymptotes lines $y = \pm\frac{b}{a}x$

$$y = \pm\frac{2}{3}x$$

2) $\frac{y^2}{9} - \frac{x^2}{4} = 1$

Solution:-

$$a^2 = 9 \Rightarrow a = \pm 3$$
$$b^2 = 4 \Rightarrow b = \pm 2$$
$$c^2 = a^2 + b^2$$
$$c^2 = a^2 + b^2 \Rightarrow c^2 = 9 + 4 \Rightarrow c^2 = 13 \Rightarrow c = \pm\sqrt{13}$$

The center (0,0)

The vertices $(0, \pm a) = (0, \pm 3)$

The foci $(0, \pm c)) = (0, \pm\sqrt{13})$

Ends of the conjugate axis $(\pm b, 0) = (\pm 2, 0)$

Asymptotes lines $y = \pm\frac{a}{b}x$

$$y = \pm\frac{3}{2}x$$

3) $x^2 - y^2 = 16$

Solution:-

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

$$a^2 = 16 \Rightarrow a = \pm 4$$
$$b^2 = 16 \Rightarrow b = \pm 4$$
$$c^2 = a^2 + b^2$$

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 16 \Rightarrow c^2 = 32 \Rightarrow c = \pm\sqrt{32}$$

The center (0,0)

The vertices $(\pm a, 0) = (\pm 4, 0)$

The foci $(\pm c, 0)) = (\pm\sqrt{32}, 0)$

Ends of the conjugate axis $(0, \pm b) = (0, \pm 4)$

Asymptotes lines $y = \pm\frac{b}{a}x$

$$y = \pm\frac{4}{4}x \qquad \Rightarrow \qquad y = \pm x$$

4) $4y^2 - 4x^2 = 1$

Solution:-

$$\frac{y^2}{\quad} - \frac{x^2}{\quad} = 1$$

$$a^2 = \ \Rightarrow a = \pm$$

$$b^2 = \ \Rightarrow b = \pm$$

$$c^2 = a^2 + b^2$$

$$c^2 = a^2 + b^2 \Rightarrow c^2 = + \Rightarrow c^2 = = \Rightarrow c = \pm$$

The center (0,0)

The vertices $(0, \pm a) = (0, \pm)$

The foci $(0, \pm c)) = (0, \pm)$

Ends of the conjugate axis $(\pm b, 0) = (\pm, 0)$

Asymptotes lines $y = \pm\frac{a}{b}x$

$$y = \pm — x$$