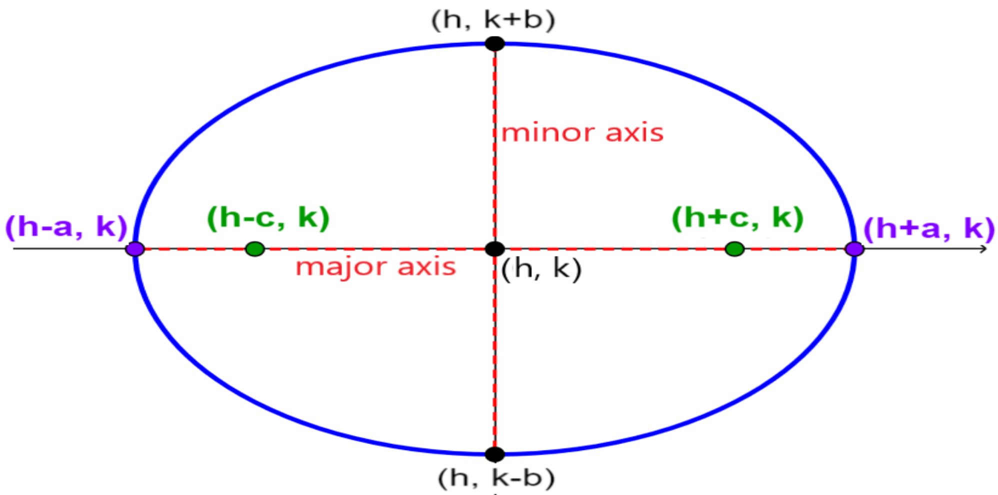
Lec(8)

The Ellipse with center (h,k)

case (1)

The equation of the ellipse with center (h,k) and paralled to the x-axis is



$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1$$

$$x' = x - h \qquad y' = y - k$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

The equation of the ellipse with center (h,k)

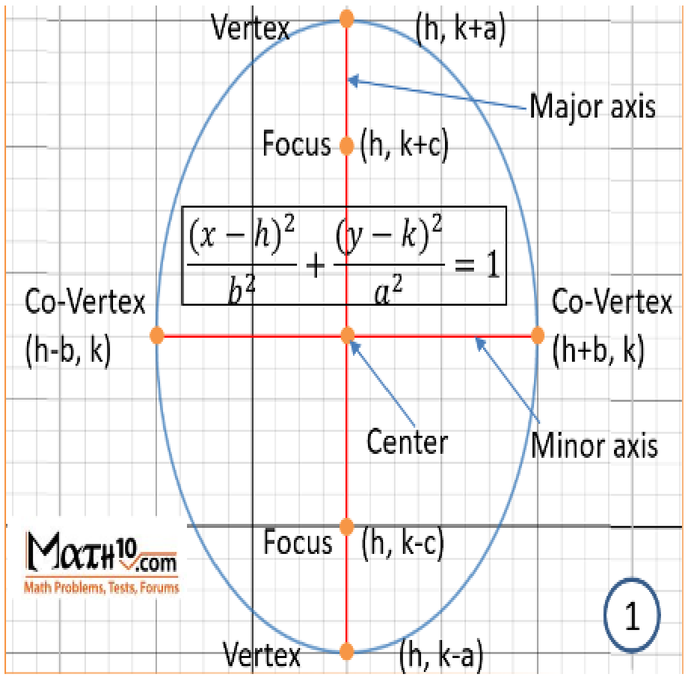
The vertices (h$\pm$a,k)

The foci (h$\pm$c,k)

the ends of the minor axis (h,k$\pm$b) is

the equation of directrices $\boldsymbol{x = h \pm \frac{a^2}{c}}$

case (2)



The equation of the ellipse with center (h,k) and paralled to the y-axis is

$$\frac{y'^2}{a^2} + \frac{x'^2}{b^2} = 1$$

$$x' = x - h \qquad\qquad y' = y - k$$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

The equation of the ellipse with center (h,k)

The vertices (h,k$\pm$a)

The foci (h, k $\pm$c)

the ends of the minor axis (h$\pm$b,k) is

the equation of directrices $y = k \pm \frac{a^2}{c}$

$$c^2 = a^2 - b^2$$

Example:-

Find the equation of the ellipse

1) **the vertices (-1,4)(3,4) , and the ends of the minor axis (1,3)(1,5)**

Solution:-

$$2a = |-1-3| = 4 \quad \Rightarrow \quad a = 2$$

$$2b = |3-5| = 2 \;\Rightarrow\; b = 1$$

$$(h,k) = \left(\frac{-1+3}{2}, \frac{4+4}{2}\right) = (1,4)$$

$$or\ (h,k) = \left(\frac{1+1}{2}, \frac{3+5}{2}\right) = (1,4)$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-1)^2}{4} + \frac{(y-4)^2}{1} = 1$$

2) the vertices (-5,11)(-5,-3) , and the foci(-5,6)(-5,2)

Solution:-

$$2a = |11 - -3| = 14 \quad \Rightarrow \quad a = 7$$

$$2c = |6 - 2| = 4 \Rightarrow \quad c = 2$$

$$c^2 = a^2 - b^2$$

$$4 = 49 - b^2 \quad \Rightarrow \quad b^2 = 49 - 4 = 45$$

$$(h, k) = \left(\frac{-5 + \; -5}{2}, \frac{11 + -3}{2}\right) = (-5, 4)$$

$$or\ (h, k) = \left(\frac{-5 + \; -5}{2}, \frac{6 + 2}{2}\right) = (-5, 4)$$

$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$

$$\frac{(y - 4)^2}{49} + \frac{(x + 5)^2}{45} = 1$$

3) the center (3,-2),foci (3,1) , (3,-5) and the ends of the minor axis (0,-2)(6,-2)1

Solution:-

$$2c = |1 - -5| = 6 \quad \Rightarrow \quad c = 3$$

$$2b = |0 - 6| = 6 \Rightarrow \quad b = 3$$

$$c^2 = a^2 - b^2$$

$$9 = a^2 - 9 \quad \Rightarrow \quad a^2 = 9 + 9 = 18$$

$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$

$$\frac{(y + 2)^2}{18} + \frac{(x - 3)^2}{9} = 1$$

4) the vertices (2,-9)(2,1) , and the length of the minor axis equal 6 units

Solution:-

$$2a = |-9 - 1| = 10 \;\Rightarrow\; a = 5$$

$$2b = 6 \;\Rightarrow\; b = 3$$

$$(h, k) = \left(\frac{2+2}{2}, \frac{-9+1}{2}\right) = (2\,, -4)$$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y+4)^2}{25} + \frac{(x-2)^2}{9} = 1$$

Example (2):-

Find the center, vertices, foci, ends of the minor axis, and the equation of directices of the following equation

1) $4x^2 - 8x + y^2 + 4y - 8 = 0$

Solution:-

$$4x^2 - 8x + y^2 + 4y - 8 = 0$$
$$4(x^2 - 2x) + y^2 + 4y - 8 = 0$$
$$4(x^2 - 2x + 1 - 1) + y^2 + 4y + 4 - 4 - 8 = 0$$
$$4[(x-1)^2 - 1] + (y+2)^2 - 12 = 0$$
$$4(x-1)^2 - 4 + (y+2)^2 - 12 = 0$$
$$4(x-1)^2 + (y+2)^2 = 16 \qquad \div 16$$

$$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{16}=1$$

$$a^2=16 \quad \Rightarrow \quad a=\pm 4$$

$$b^2=4 \quad \Rightarrow \quad b=\pm 2$$

$$c^2=a^2-b^2=16-4=12 \ \Rightarrow c=\pm\sqrt{12}$$

The center (h,k) = (1,-2)

The vertices $(h,k\pm a)=(1,-2\pm 4) \ \Rightarrow (1,2),(1,-6$

The foci $(h,k\pm c)=\left(1,-2\pm\sqrt{12}\right) \ \Rightarrow$

$$\left(1,-2+\sqrt{12}\right),\left(1,-2-\sqrt{12}\right)$$

The ends of minor axis $(h\pm b,k) \ \Rightarrow (1\pm 2\ ,-2) \ \Rightarrow$
$(3,-2).(-1,-2)$

The equation of directrix $y=k\pm\frac{a^2}{c}= \ -2\pm\ \frac{16}{\sqrt{12}}$

2)

$$\frac{(x+2)^2}{10}+(y+3)^2=1$$

3)

$$\frac{(x+3)^2}{100}+\frac{(y-2)^2}{36}=1$$