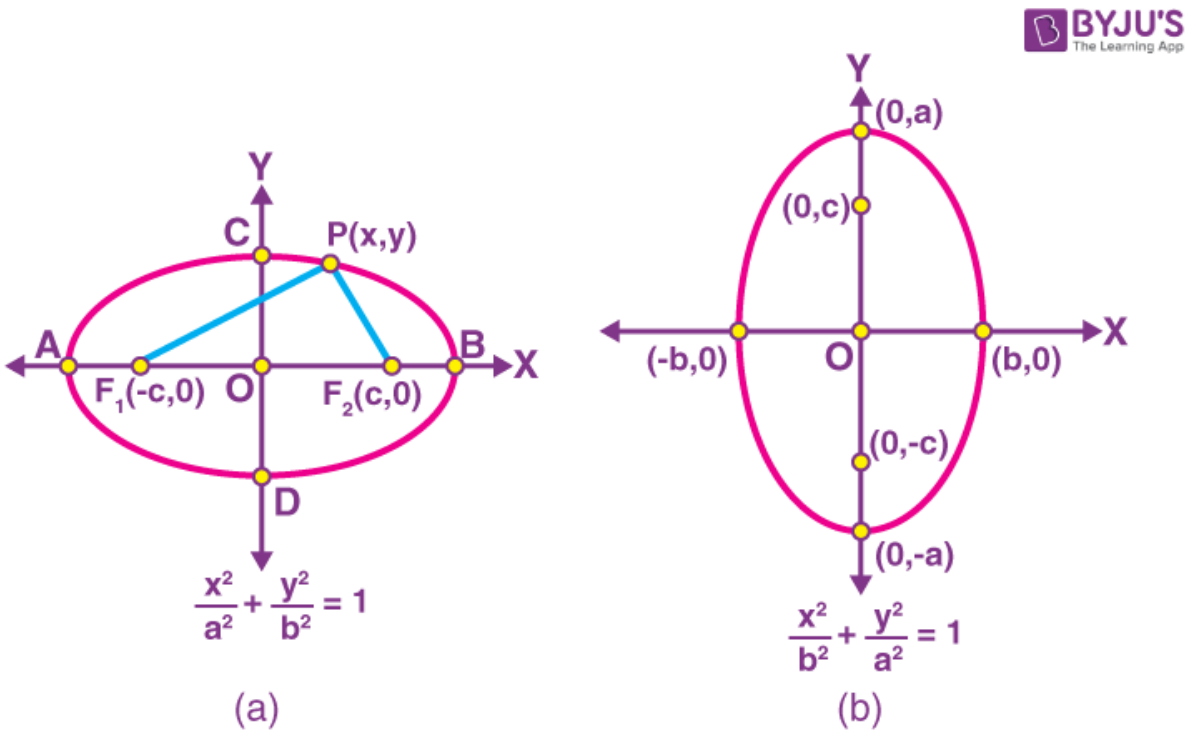
Lec(7)

The Ellipse

Definition:-

The ellipse is the set of all points P in the plane such that the sum of the distance of P from two fixed points F_1 and F_2 on the plane is constant equal 2a .“ each of two fixed points are called foci “ .



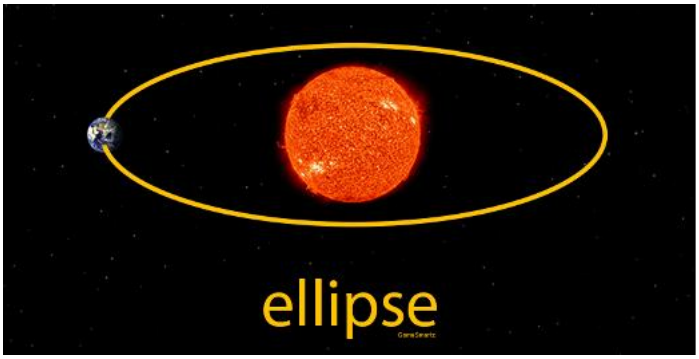
The line $\overline{VV'}$ is called the major axis and its length equal 2a.

The line $\overline{MM'}$ is called the minor axis and its length equal 2b.

The points V and V’ point are called the ends of the major axis or vertices.

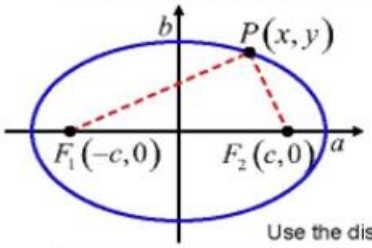
The points M and M' are called the ends of minor axis

The points F and F' are called foci.



Appendix A – deriving the equation of an Ellipse from existing definitions[2].

Deriving the Equation of an Ellipse



Something to notice:

$$b^2 = a^2 - c^2$$

Equation for the two dashed distances:

$$PF_1 + PF_2 = 2a$$

Use the distance formula:

$$\sqrt{(x+c)^2+(y-0)^2} + \sqrt{(x-c)^2+(y-0)^2} = 2a$$

$$\sqrt{(x-c)^2+y^2} = 2a - \sqrt{(x+c)^2+y^2}$$

$$x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + x^2 + 2cx + c^2 + y^2$$

Deriving the Equation of an Ellipse

$$x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + x^2 + 2cx + c^2 + y^2$$

$$a\sqrt{(x+c)^2+y^2} = a^2 + cx$$

$$a^2\left(x^2 + 2cx + c^2 + y^2\right) = a^4 + 2a^2cx + c^2x^2$$

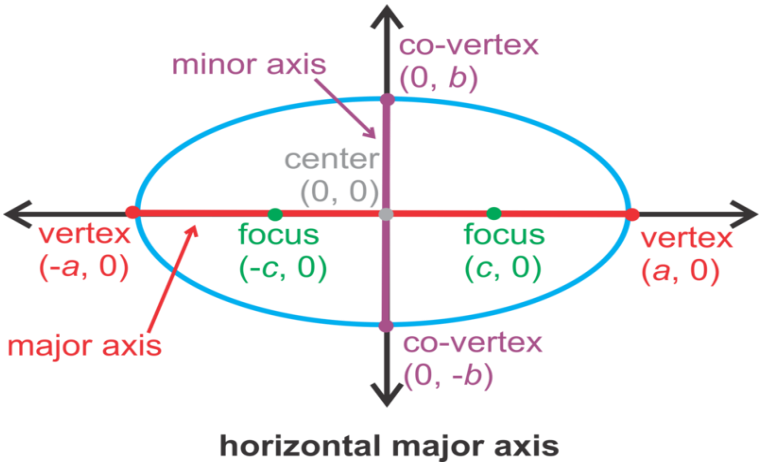
$$a^2x^2 + 2a^2cx + a^2c^2 + a^2y^2 = a^4 + 2a^2cx + c^2x^2$$

$$\left(a^2 - c^2\right)x^2 + a^2y^2 = a^2\left(a^2 - c^2\right)$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

1)

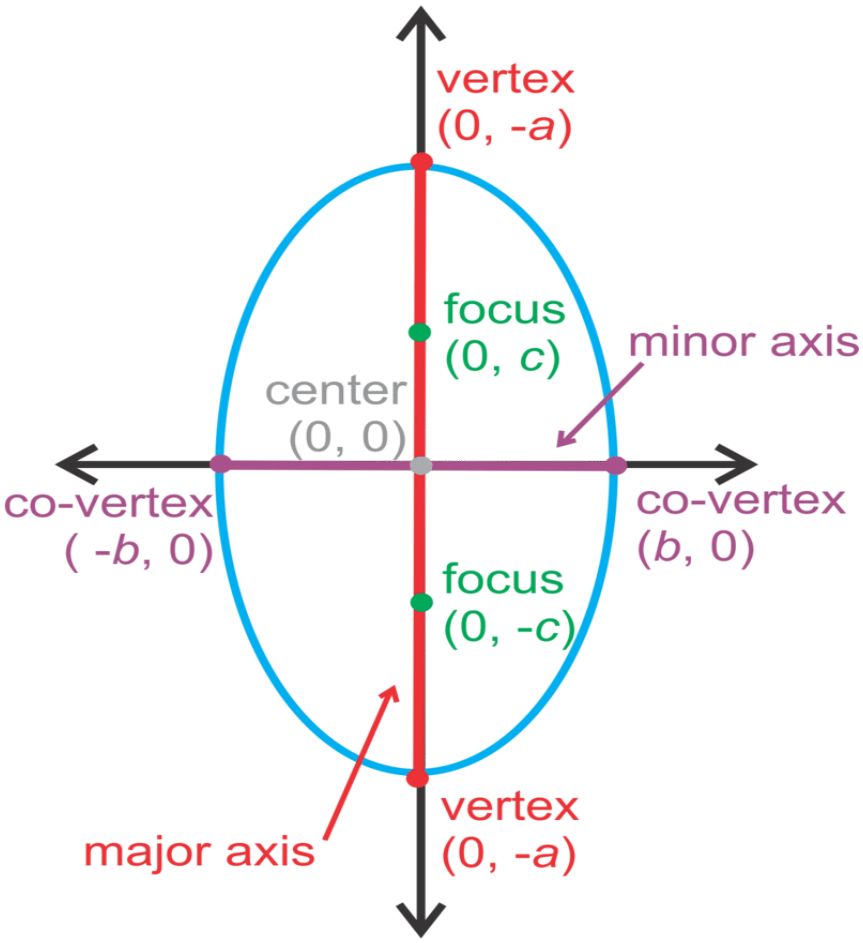


horizontal major axis

The equation of the ellipse with center (0,0) , the major axis on the x-axis , vertices (a,0),(-a,0) , foci (c,0),(-c,0) and the ends of the minor axis (0,b),(0,-b) is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

2)



vertical major axis

The equation of the ellipse with center (0,0) , the major axis on the y-axis , vertices (0,a),(0,-a) , foci (0,c),(0,-c) and the ends of the minor axis (b,0),(-b,0) is

$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$$

Example:-

Find the equation of the ellipse

1) the foci $(\pm 2, 0)$, and the length of the minor axis equal 6 units

Solution:-

$$c = \pm 2$$

$$2b = 6 \Rightarrow \frac{2b}{2} = \frac{6}{2} \Rightarrow b = 3$$

$$c^2 = a^2 - b^2$$

$$4 = a^2 - 9 \Rightarrow a^2 = 4 + 9 \Rightarrow a^2 = 13$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{13} + \frac{y^2}{9} = 1$$

2) Foci $(0, \pm 2)$, vertices " ends of the major axis =$(0, \pm 4)$

Solution:-

$$c = \pm 2 \quad , \quad a = \pm 4$$

$$c^2 = a^2 - b^2$$

$$4 = 16 - b^2 \Rightarrow b^2 = 16 - 4 \Rightarrow b^2 = 12$$

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{16} + \frac{x^2}{12} = 1$$

Example (2):-

Find the center, vertices, foci, ends of the minor axis, and the length of the major axis, minor axis of the following equations

1) $\frac{x^2}{16}+\frac{y^2}{25}=1$

Solution:-

$$a^2=25 \Rightarrow a=\pm 5$$
$$b^2=16 \Rightarrow b=\pm 4$$
$$c^2=a^2-b^2$$
$$c^2=25-16 \Rightarrow c^2=9 \Rightarrow c=\pm 3$$

The center (0,0)

The vertices $(0,\pm a)=(0,\pm 5)$

The foci $(0,\pm c)=(0,\pm 3)$

2) $\frac{x^2}{25}+\frac{y^2}{16}=1$

Solution:-

$$a^2=25 \Rightarrow a=\pm 5$$
$$b^2=16 \Rightarrow b=\pm 4$$
$$c^2=a^2-b^2$$
$$c^2=25-16 \Rightarrow c^2=9 \Rightarrow c=\pm 3$$

The center (0,0)

The vertices $(\pm a,0)=(\pm 5,0)$

The foci $(\pm c,0)=(\pm 3,0)$

3) $\boldsymbol{64x^2 + 100y^2 = 6400}$

Solution:-

$$\boldsymbol{a^2 = \ \Rightarrow a = \pm}$$
$$\boldsymbol{b^2 = \ \Rightarrow b = \pm}$$
$$\boldsymbol{c^2 = a^2 - b^2}$$
$$\boldsymbol{c^2 = \ - \ \Rightarrow c^2 = \Rightarrow c = \pm}$$

The center (0,0)

The vertices (,) = (,)

The foci (.) = (,)

4) $\boldsymbol{x^2 + 4y^2 = 1}$

5) $\boldsymbol{4x^2 + 5y^2 = 20}$