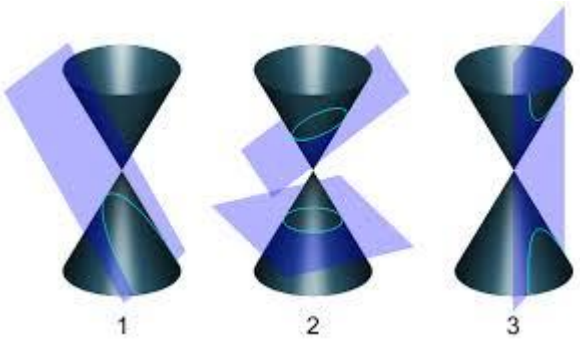
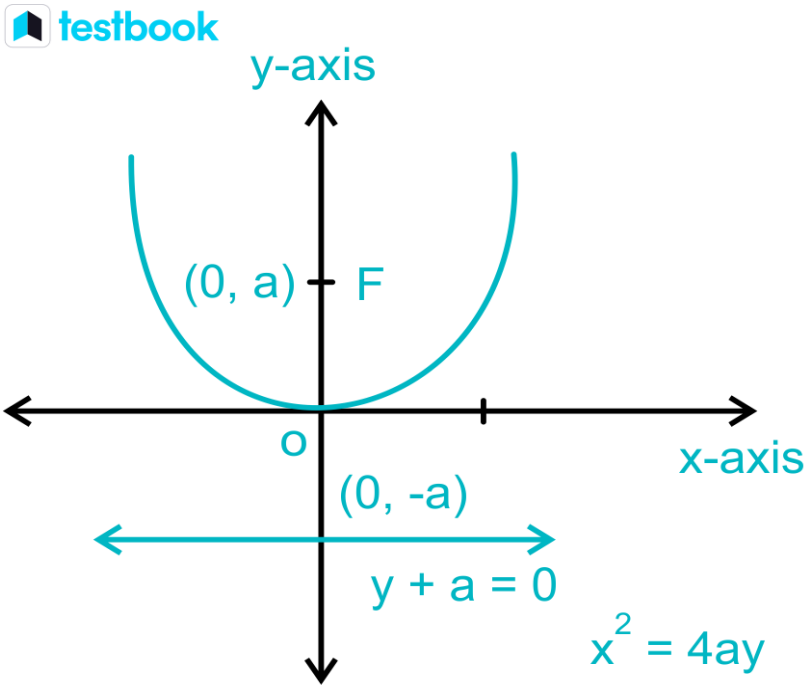


The parabola

Case 3:

The parabola with vertex (0,0), focus (0,a) the equation of directix $y = -a$ and open in the positive y-direction.

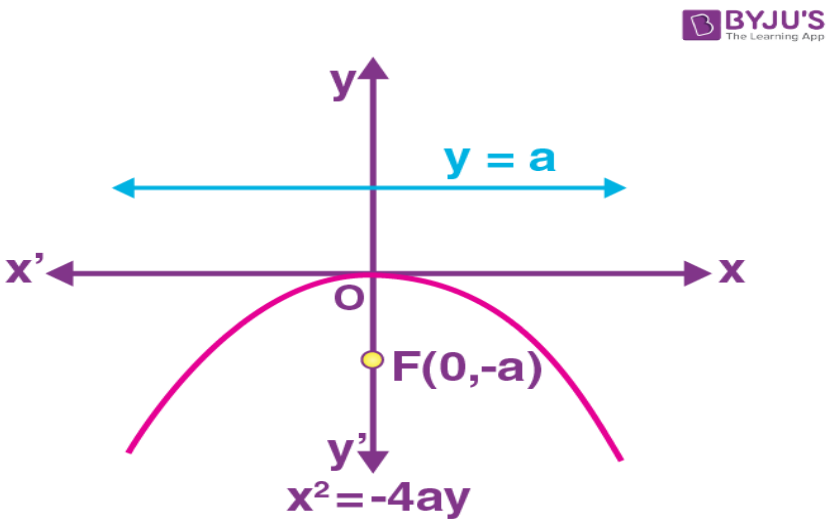
$$x^2 = 4ay$$



Case 2:

The parabola with vertex at the origin (0,0) , focus (0,-a), the equation of directrix y = a , and open in the negative y-direction from the definition

$$x^2 = -4ay$$



Example:-

i) Find the equation of a parabola if:-

1) **Vertex (0,0) and focus (0,6)**
   Solution

$$a = 6$$

$$x^2 = 4ay \quad \Rightarrow \quad x^2 = (4)(6)y$$

$$x^2 = 24y$$

2) **Vertex (0,0) and the equation of the directrix** $y = -3$

Solution

$$a = 3$$

$$x^2 = 4ay \quad \Rightarrow \quad x^2 = 12y$$

3) **Vertex (0,0) and focus (0,-2)**

Solution

$$a = 2$$

$$x^2 = -4ay \quad \Rightarrow \quad y^2 = -8x$$

4) **vertex (0,0) and the equation of the directrix** $y = 5$

Solution

$$a = 5$$

$$x^2 = -4ay \quad \Rightarrow \quad x^2 = -20y$$

ii) find the coordinates of vertex, focus, and the equation of the directrix of the following equation

1) $x^2 = 36y$

Solution

$$x^2 = 4ay \Rightarrow 4a = 36 \Rightarrow \frac{4a}{4} = \frac{36}{4} \Rightarrow a = 9$$

Vertex (0,0)

Focus(0,a)=(0,9)

The equation of the directrix

$$y = -a$$

$$y = -9$$

2) $x^2 = -y$

Solution

$$x^2 = -4ay \Rightarrow -4a = -1 \Rightarrow a = \frac{-1}{-4} = \frac{1}{4}$$

Vertex (0,0)

Focus(0,-a)=$(0, -\frac{1}{4})$

The equation of the directrix

$$y = a \quad \Rightarrow \quad y = \frac{1}{4}$$

3) $x^2 = -\frac{1}{4}y$

Solution

$$x^2 = -4ay \Rightarrow -4a = \frac{-1}{4} \Rightarrow \frac{-4a}{-4} = \frac{-1}{4}.\frac{1}{-4} \Rightarrow a = \frac{1}{16}$$

Vertex (0,0)

Focus(0,-a)=(0,$\frac{-1}{16}$)

The equation of the directrix

$$y = a$$

$$y = \frac{1}{16}$$

4) $x^2 = 9y$

Solution

$$x^2 = 4ay \Rightarrow 4a = 9 \Rightarrow \frac{4a}{4} = \frac{9}{4} \Rightarrow a = \frac{9}{4}$$

Vertex (0,0)

Focus$(0, a) = (0, \frac{9}{4})$

The equation of the directrix

$$y = -a$$

$$y = -\frac{9}{4}$$

The parabola with vertex (h,k)

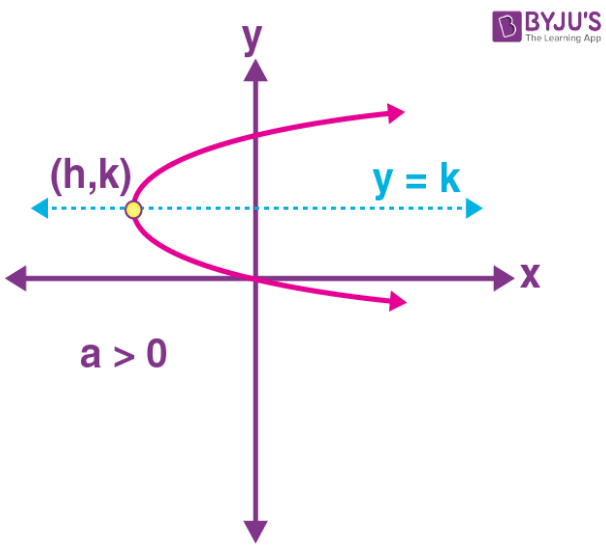
Case (1):-

The parabola with vertex (h,k) and open in the positive x-direction the equation of it with respect to

$$(y-k)^2 = 4a(x-h)$$

This is a parabola with vertex (h,k) focus(h+a,k) and the equation of directrix is $x = h - a$

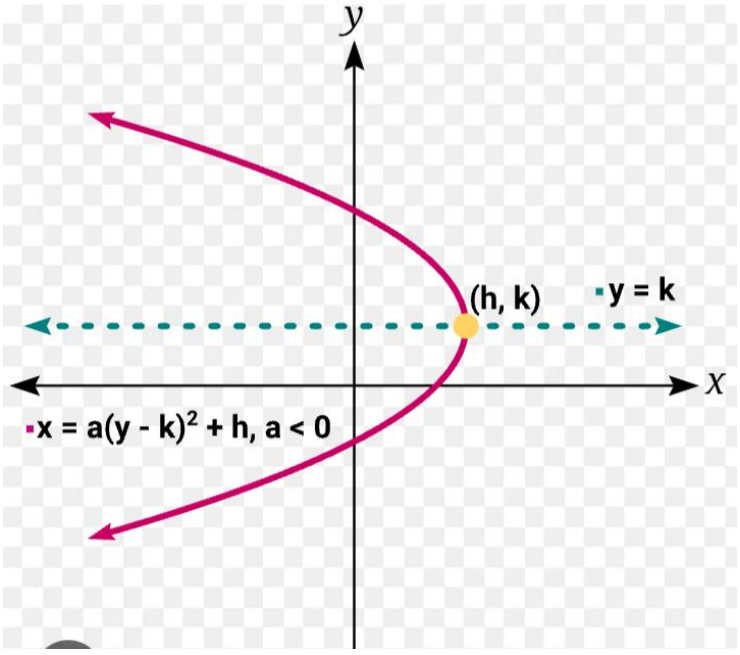
Parabola axis is parallel to x-axis



Case 2 :-

Parabola axis is parallel to x-axis , vertex (h,k) open in the negative x-direction , focus $(h-a, k)$ and the equation of directrix $x = h + a$

$$(y-k)^2 = -4a(x-h)$$



Example:-

Find the equation of parabola if

1) The vertex(2,3)and the focus (7,3)

Solution

$$(y-k)^2 = 4a(x-h)$$

$$(h,k) = (2,3)$$

$$a = 5$$

$$(y-3)^2 = 20(x-2)$$

The diretrix $x = h - a$

$$x = 2 - 5$$

$$x = -3$$

$$y = 3$$

2) The vertex(-3,-1)and the focus (-5,-1)

Solution

$$(y-k)^2 = -4a(x-h)$$

$$(h,k) = (-3,-1)$$

$$a = 2$$

$$(y--1)^2 = -(4)(2)(x--3)$$

$$(y+1)^2 = -8(x+3)$$

The diretrix $x = h + a$

$$x = -3 + 2$$

$$x = -1$$

$$y = -1$$