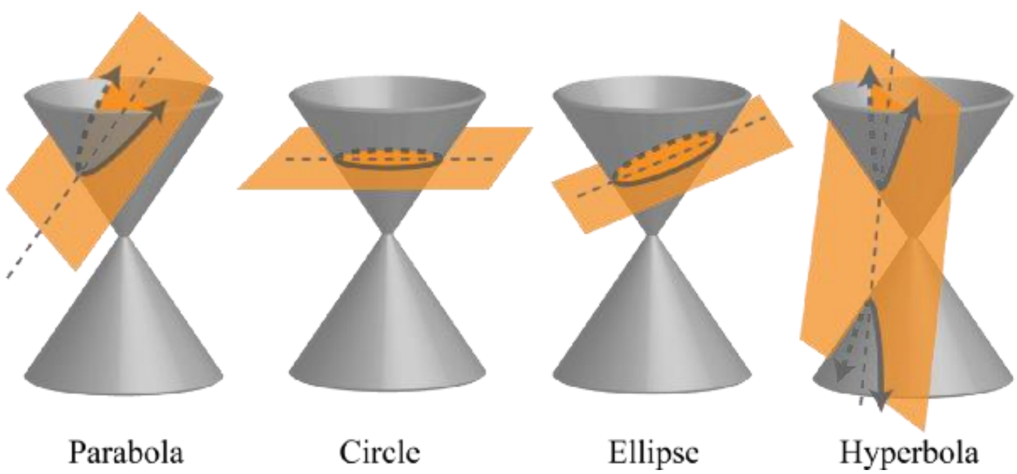
Conic sections

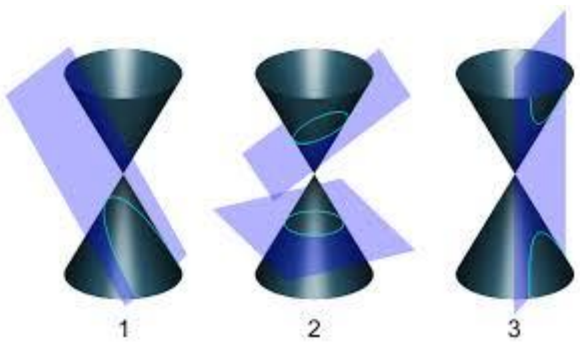
Definition



The conic section is the set of all points in the plane such that the ratio between their distance from a fixed point and their distance from a fixed line is always constant, the fixed point is called the focus, the fixed line is called the directrix , and the constant ratio is called the eccentricity , denoted by e.

The conic sections depend on the value of e as follow:-

1) **If $e = 1$, then the conic section is called the parabola.**
2) **If $e < 1$, then the conic sections is called an ellipse.**
3) **If $e > 1$, then the conic sections is called hyperbola.**



The parabola

Definition:-

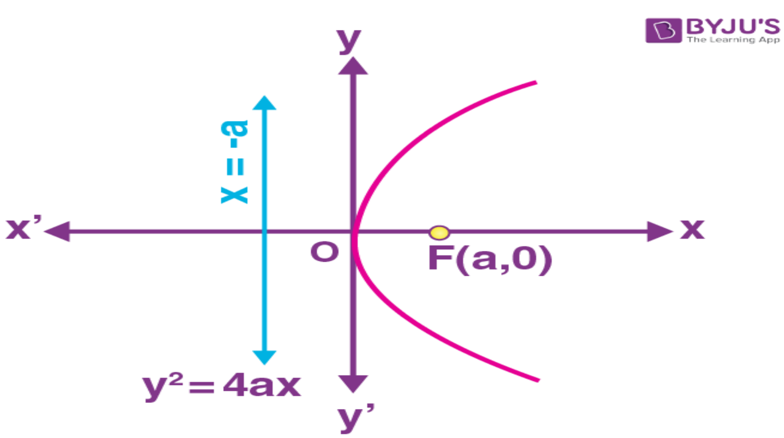
The parabola is the set of all points in a plane which for the same distance from a fixed point and a fixed line of the plane of the plane where the fixed point is called the focus and the fixed line is called the directrix.

Case 1:

Let p(x,y) be an point on the plane from the definition $\overline{PF} = \overline{pq}$

We get

$$\boxed{y^2 = 4ax}$$



This is the equation of parabola with vertex (0,0), focus (a,0),the equation of directix x = -a and open in the positive x-direction.

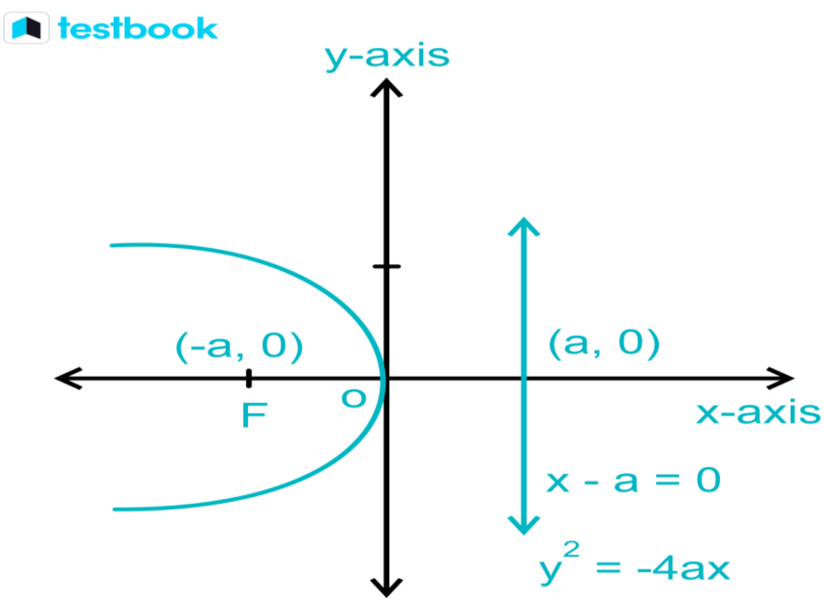
Case 2:

The parabola with vertex at the origin (0,0) , focus (-a,0), the equation of directrix x = a , and open in the negative x-direction from the definition

$$\overline{PF} = \overline{pq}$$

We get

$$\boxed{y^2 = -4ax}$$



Example:-

i) Find the equation of a parabola if:-

1) **Vertex (0,0) and focus (3,0)**
   Solution

$$a = 3$$
$$y^2 = 4ax \quad \Rightarrow \quad y^2 = 12x$$

2) Vertex (0,0) and the equation of the directrix x=2

Solution

$$a = 2$$

$$y^2 = -4ax \quad \Rightarrow \quad y^2 = -8x$$

3) Vertex (0,0) and focus (-5,0)

Solution

$$a = 5$$

$$y^2 = -4ax \quad \Rightarrow \quad y^2 = -20x$$

4) vertex (0,0) and the equation of the directrix x=-1

Solution

$$a = 1$$

$$y^2 = 4ax \quad \Rightarrow \quad y^2 = 4x$$

ii) find the coordinates of vertex, focus, and the equation of the directrix of the following equation

1) $y^2 = 16x$

Solution

$$y^2 = 4ax \Rightarrow 4a = 16 \Rightarrow \frac{4a}{4} = \frac{16}{4} \Rightarrow a = 4$$

Vertex (0,0)

Focus(a,0)=(4,0)

The equation of the directrix

$$x = -a$$

$$x = -4$$

2) $y^2 = x$

Solution

$$y^2 = 4ax \Rightarrow 4a = 1 \Rightarrow a = \frac{1}{4}$$

Vertex (0,0)

Focus(a,0)=($\frac{1}{4}$,0)

The equation of the directrix

$$x = -a \quad \Rightarrow \quad x = -\frac{1}{4}$$

3) $y^2 = -4x$

Solution

$$y^2 = -4ax \Rightarrow -4a = -4 \Rightarrow \frac{-4a}{-4} = \frac{-4}{-4} \Rightarrow a = 1$$

Vertex (0,0)

Focus(-a,0)=(-1,0)

The equation of the directrix

$$x = -a$$
$$x = 1$$

4) $y^2 = -6x$

Solution

$$y^2 = -4ax \Rightarrow -4a = -6 \Rightarrow \frac{-4a}{-4} = \frac{-6}{-4} \Rightarrow a = \frac{3}{2}$$

Vertex (0,0)

Focus$(-a, 0) = (\frac{-3}{2}, 0)$

The equation of the directrix

$$x = -a$$
$$x = -\frac{3}{2}$$