Lecture (3)

Analytical Analysis

The Circle

Definition:

A circle is the sea of all points in a plane that are fixed distance from a fixed point called its center, the fixed distance is called the radius of the circle.

Circles equations:-

1) $x^2 + y^2 = r^2$

The center(0,0) and the radius is r.

2) $(x - h)^2 + (y - k)^2 = r^2$

The center (h,k) and the radius is r

3) $x^2 + y^2 + 2hx + 2ky + c = 0$

The center = $\left(\frac{-cofficient\ of\ x}{2} . \frac{-cofficient\ of\ y}{2}\right)$

(m,n) = $(\frac{-2h}{2} . \frac{-2k}{2})$ **= (-h,-k)**

The radius r = $\sqrt{m^2 + n^2 - c}$

The conditions of circles equations:

1) the cofficient of x^2 **= the cofficient of** y^2 **= 1.**

2) there is no xy in the equation of circle.

Example

Find the center and the radius of the following circles equations

1) $x^2 + y^2 = 16$

Solution

The center(0,0) , the radius r=4

2) $(x-4)^2 + (y-1)^2 = 9$

The center (4,1) , r=3

3) $(x+3)^2 + y^2 = 12$

The center (-3,0) , r=$\sqrt{12}$ **.**

4) $2x^2 + 2y^2 + 4x - 8y + 2 = 0$

Divided by 2

$x^2 + y^2 + 2x - 4y + 1 = 0$

The center (m,n) = $(\frac{-2}{2}.\frac{--4}{2}) = (-1.2)$

$$r = \sqrt{m^2 + n^2 - c} = \sqrt{(-1)^2 + (2)^2 - 1}$$
$$= \sqrt{1 + 4 - 1} = \sqrt{4} = 2$$

Example:

Write the equation of a circle of radius r=$\sqrt{7}$ **with center (-2,3)**

Solution

$(x-h)^2 + (y-k)^2 = r$

$(x--2)^2 + (y-3)^2 = \left(\sqrt{7}\right)^2$

$$(x+2)^2+(y-3)^2=7$$

Example :

Find the center and the radius of a circle

$$x^2+y^2-4x+2y-11=0$$

solution

$$x^2-4x+4-4+y^2+2y+1-1=0$$

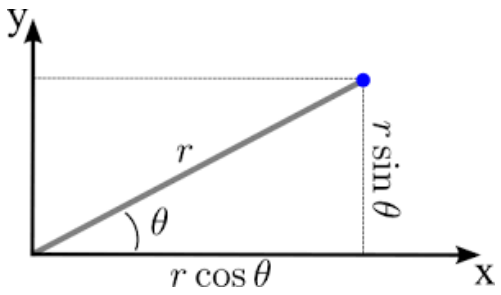
$$(x-2)^2+(y+1)^2=16$$

Center(2,-1) , radius = 4

Polor Coordinates

Let o is a fixed point called the origin point astraight line is extended , from 0 to the point A , OA is called polor axis.

Let p is a point which for a distance r from the point 0 (r,θ) is apair of polor coordinate , used to determine a cartesin point , where r is the distance from a cartesin point p to the origin point 0 , and θ is the angle between OA and OP

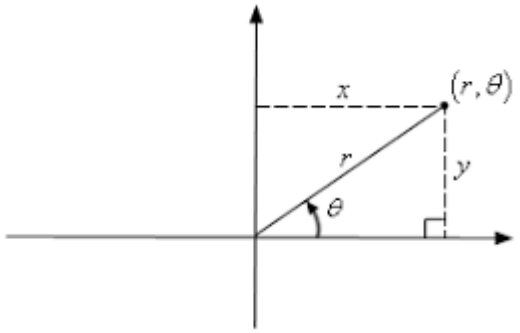


the relation between cartesian coordinates and polor coordinates

let p is a poin

t has the coordinates (x,y) and the polor

coordinates($r.\theta$)



$$sin(\theta) = \frac{y}{r} \qquad y = r\ sin(\theta)$$

$$cos(\theta) = \frac{x}{r} \qquad x = r\ cos(\theta)$$

$$r = \sqrt{x^2 + y^2}$$

$$tan(\theta) = \frac{y}{x} \quad \Longrightarrow \quad \theta = tan^{-1}(\frac{y}{x})$$

Example:

Plot the following points in the polar coordinate

1) $(2,\frac{\pi}{6})$ **2)** $(5,\frac{3\pi}{4})$ **3)** $(-2,\frac{2\pi}{3})$ **4)** $(1,\frac{-\pi}{4})$

Example:

Find the cartesian coordinates of the following points

1) $\left(4.\frac{\pi}{6}\right)$ **2)** $(2,\frac{3\pi}{4})$ **3)** $(3,\frac{-\pi}{6})$

Solution

1) $x = r\,cos(\theta)$ $x = 4cos\left(\frac{\pi}{6}\right)$ $x = 4($

$y = rsin(\theta)$ $y = 4sin\left(\frac{\pi}{6}\right)$ $y = 4($

$(x.y) = (\ .\ )$

2)

3)

Example :

Find the polar coordinate of the following points

1) $(\sqrt{3}.1)$ **2) (1.-1)** **3) (-2,-2)**

Solution

$$\mathbf{1)}\ r = \sqrt{x^2 + y^2} = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\theta = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30\ = \frac{\pi}{6}$$

2)

3)