Lecture (2)

Analytical Geometry

The straight line

The general form of the straight line equation is

$$Ax + By + c = 0$$

The gradient(slope) of the straight line is denoted by m

m = $\frac{-\ coefficient\ of\ x}{coefficient\ of\ y}$ = $\frac{-A}{B}$

m = tan l

m = $\frac{y_2 - y_1}{x_2 - x_1}$

Some forms of the straight line equation :

$$y = mx + b$$

2) the intercepts equation of the straight line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Such that a is the intercept on the x-axis and b is the intercept on the y-axis.

3) the equation of a straight line with given gradient passing through a given poin A($x_1 . y_1$) is

$$y - y_1 = m(x - x_1)$$

4) the equation of a straight line passing through the two points ($x_1 . x_2$) and ($y_1 . y_2$) is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel and Perpendicular lines

The lines are parallel if and only if they have equal slopes $(m_1 = m_2)$

The lines are perpendicular if and only if $m_1 m_2 = -1$

Example(1) :

Find the equation of the lines described below

1) gradient 5 , y-intercepts 3

Solution

$$y = mx + b \implies y = 5x + 3$$

2) gradient 3 passing through the origion

Solution

$$y = mx \implies y = 3x$$

3) gradient $\frac{-3}{4}$, passing through (1,-2)

Solution

$$(y - y_1) = m(x - x_1)$$

$$y - -2 = \frac{-3}{4}(x - 1)$$

$$4y + 8 = -3x + 3$$

$$4y + 3x + 5 = 0$$

Example(2)

Find the equations of the lines through the given point

1) parallel

2) perpendicular

To the given line

1) $4x - 8y = 5 \quad . (-1. -3)$

2) $\sqrt{2}\, y + 11 = 0 \ . (-2. 0)$

Solution

$$m = \frac{-cofficient\ of\ x}{cofficient\ of\ y} = \frac{-4}{-8} = \frac{1}{2} \quad m_1\ parallei$$

$$y - y_1 = m_1(x - x_1)$$

$$n_1 = \frac{-1}{m} = -2 \quad perpendicular$$

$$y - y_1 = m_{1(x-x_1)}$$

$$y + 3 = -2(x + 1) \implies y + 3 = -2x - 2$$

$$2x + y + 5 = 0$$

2) $m = \frac{-0}{\sqrt{2}} = 0$

$$(y - y_{1)} = m(x - x_1)$$

$$y - 0 = 0(x + 2) \implies y = o$$

$$m_1 = \frac{-1}{m} = \frac{-1}{0}$$

$$y - y_1 = m_1(x - x_1)$$

$$y - o = \frac{-1}{0}(x + 2) \longrightarrow -(x + 2) = 0 \ . x + 2 = 0$$

Example(3)

Find the equation of the lines described below

1) gradient -2 , passing through (2,0)

Solution

$$(y - y_1) = m(x - x_1)$$

$$y - 0 = -2(x - 2) \implies y = -2 + 4$$

2) gradient 0 , passing through (-1,2)

Solution

$$(y - y_1) = m(x - x_1)$$

$$y - 2 = 0(x + 1) \implies y - 2 = 0 . y = 2$$

3) passing through (-2,3) and (2,-5)

Solution

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - 3}{x + 2} = \frac{-5 - 3}{2 + 2} = \frac{-8}{4} = -2$$

$$y - 3 = -2(x + 2) \implies y - 3 = -2x - 4 \implies 2x + y + 1 = 0$$

4) with intercepts $\frac{4}{5}$ and $\frac{-3}{4}$

Solution

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{\frac{4}{5}} + \frac{y}{\frac{-3}{4}} = 1 \quad . \quad \frac{5x}{4} + \frac{4y}{-3} = 1$$

$$15x - 16y = 12$$

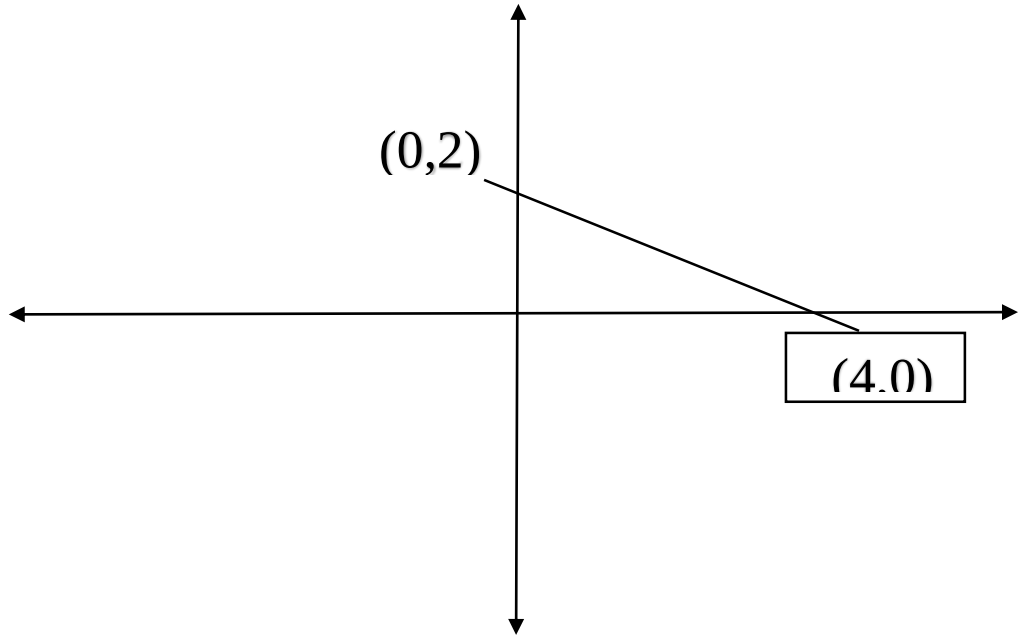
Example(4)

Graph $x + 2y + 4$

Solution

$$put \quad x + 0 \implies y = 2 \implies (0.2)$$

$$put \quad y = 0 \implies x + 4 \implies (4.0)$$



2) $\boldsymbol{y = 3x}$

Solution

$$\boldsymbol{put\ x = 0 \implies y = 0 \implies (0.0)}$$

$$\boldsymbol{put\ y = 0 \implies x = 0 \implies (0.0)}$$

$$\boldsymbol{put\ x = 1 \implies y = 3 \implies (1.3)}$$