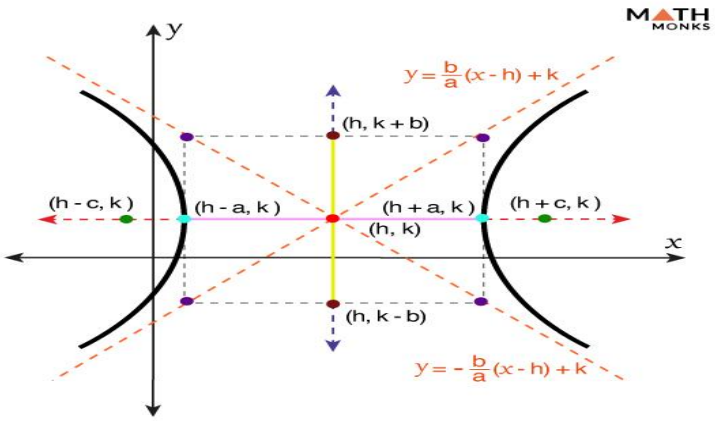
Lecture (10)

The hyperbola with center (h,k)

Case (1):-



$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

here,
a = distance from the center to a vertex
b = distance from the center to a co-vertex

The equation of a hyperbola with center (h,k) , and the focal axis is parallel the x- axis is

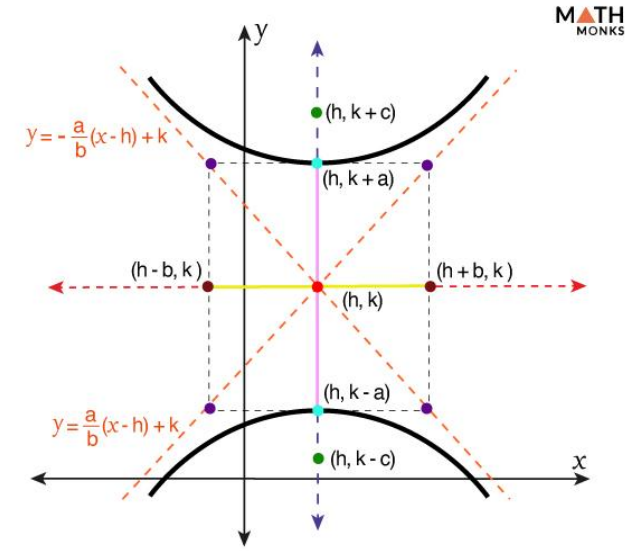
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

The equation of the hyperbola with center (h,k) , the focal axis on the x-axis , vertices $(h \pm a, k)$, foci $(h \pm c, k$ and ends of the conjugate axis $(h, b \pm k)$

Asymptotes lines $y - k = \pm \frac{b}{a}(x - h)$

The equation of directrices $x = h \pm \frac{a^2}{c}$

Case (2):-



$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

here,
a = distance from the center to a vertex
b = distance from the center to a co-vertex

The equation of a hyperbola with center (h,k) , and the focal axis is parallel the y- axis is

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The equation of the hyperbola with center (h,k) , the focal axis on the x-axis , vertices $(h, k \pm a)$, foci $(h, k \pm c)$ and ends of the conjugate axis $(h \pm b, k)$

Asymptotes lines $y - k = \pm\frac{a}{b}(x - h)$

The equation of directrices $y = k \pm \frac{a^2}{c}$

Example:-

Find the equation of the hyperbola if

1) center (2,5) , the foci $(2,3)$,(2,7) ,and vertices $(2,4)(2,6)$

Solution:-

$$2c = |3-7| = 4 \quad \Rightarrow \quad c = 2$$
$$2a = |4-6| = 2 \quad \Rightarrow \quad a = 1$$
$$c^2 = a^2 + b^2$$
$$4 = 1 + b^2 \Rightarrow b^2 = 4 - 1 \Rightarrow b^2 = 3$$

The focal axis is parallel to y-axis

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-5)^2}{1} - \frac{(x-2)^2}{3} = 1$$

2) vertices(0,2),(6,2) and foci (-1,2),(7,2)

Solution:-

$$(h,k) = \left(\frac{0+6}{2}, \frac{2+2}{2}\right) = (3,2) \quad \text{or}$$
$$(h,k) = \left(\frac{-1+7}{2}, \frac{2+2}{2}\right) = (3,2)$$
$$2a = |0-6| = 6 \quad \Rightarrow \quad a = 3$$
$$2c = |-1-7| = 8 \quad \Rightarrow \quad c = 4$$
$$c^2 = a^2 + b^2$$
$$16 = 9 + b^2 \Rightarrow b^2 = 16 - 9 \Rightarrow b^2 = 7$$

The focal axis is parallel to x-axis

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{7} = 1$$

3) center (-4,1) , one of vertices $(2, 1)$,and the length of the conjugate axis equal 8 units

Solution:-

$$2b = 8 \Rightarrow b = 4$$

$$(h, k) = (-4, 1)$$

$$(h \pm a, k) = (2, 1) \;\Rightarrow\; -4 \pm a = 2 \;\Rightarrow\; a = \pm 6$$

The focal axis is parallel to x-axis

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x+4)^2}{36} - \frac{(y-1)^2}{16} = 1$$

Example (2):-

Find the center, vertices, foci, ends of the conjugate axis, and asymptotes of the following equations

1) $\frac{(x+1)^2}{4} - \frac{(y+3)^2}{16} = 1$

Solution:-

$$a^2 = 4 \Rightarrow a = \pm 2$$

$$b^2 = 16 \Rightarrow b = \pm 8$$

$$c^2 = a^2 + b^2$$

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 16 \Rightarrow c^2 = 20 \Rightarrow c = \pm\sqrt{20}$$

The center $(h, k) = (-1, -3)$

The vertices $(h \pm a, k) = (-1 \pm 2, -3) \;\Rightarrow\; (-3, -3), (1, -3)$

The foci $(h \pm c, k) = \left(-1 \pm \sqrt{20}, -3\right) \Rightarrow \left(-1 + \sqrt{20}, -3\right), \left(-1 - \sqrt{20}, -3\right)$

Ends of the conjugate axis $(h, k \pm b) = (-1, -3 \pm 4) \Rightarrow (-1, 1), (-1, -7)$

Equation of directrices $x = h \pm \frac{a^2}{c}$

$$x = -1 \pm \frac{4}{\sqrt{20}} = -1 \pm \frac{2}{\sqrt{5}}$$

2) $9x^2 - 16y^2 - 18x - 64y - 199 = 0$

Solution:-

$$9x^2 - 18x - 16y^2 - 64y - 199 = 0$$
$$9(x^2 - 2x) - 16(y^2 + 4y) - 199 = 0$$
$$9(x^2 - 2x + 1 - 1) - 16(y^2 + 4y + 4 - 4) - 199 = 0$$
$$9((x-1)^2 - 1) - 16((y+2)^2 - 4) - 199 = 0$$
$$9(x-1)^2 - 9 - 16(y+2)^2 - 64 - 199 = 0$$
$$9(x-1)^2 - 16(y+2)^2 - 9 + 64 - 199 = 0$$
$$9(x-1)^2 - 16(y+2)^2 - 144 = 0$$
$$9(x-1)^2 - 16(y+2)^2 = 144 \qquad \div 144$$
$$\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$$

The center $(h, k) = ()$

The vertices $(h \pm a, k) =)$

The foci $(h \pm c, k) =$

Ends of the conjugate axis $(h, k \pm b) =$

Equation of directrices $x = h \pm \frac{a^2}{c}$

$$x = \pm —$$

3) $9x^2 - 4y^2 - 54x + 8y + 113 = 0$

4) $4y^2 - 9x^2 - 18x - 8y = 41$