University of Science and Technology

Faculty of Engineering

first Year - Semester (1)

physics (1)

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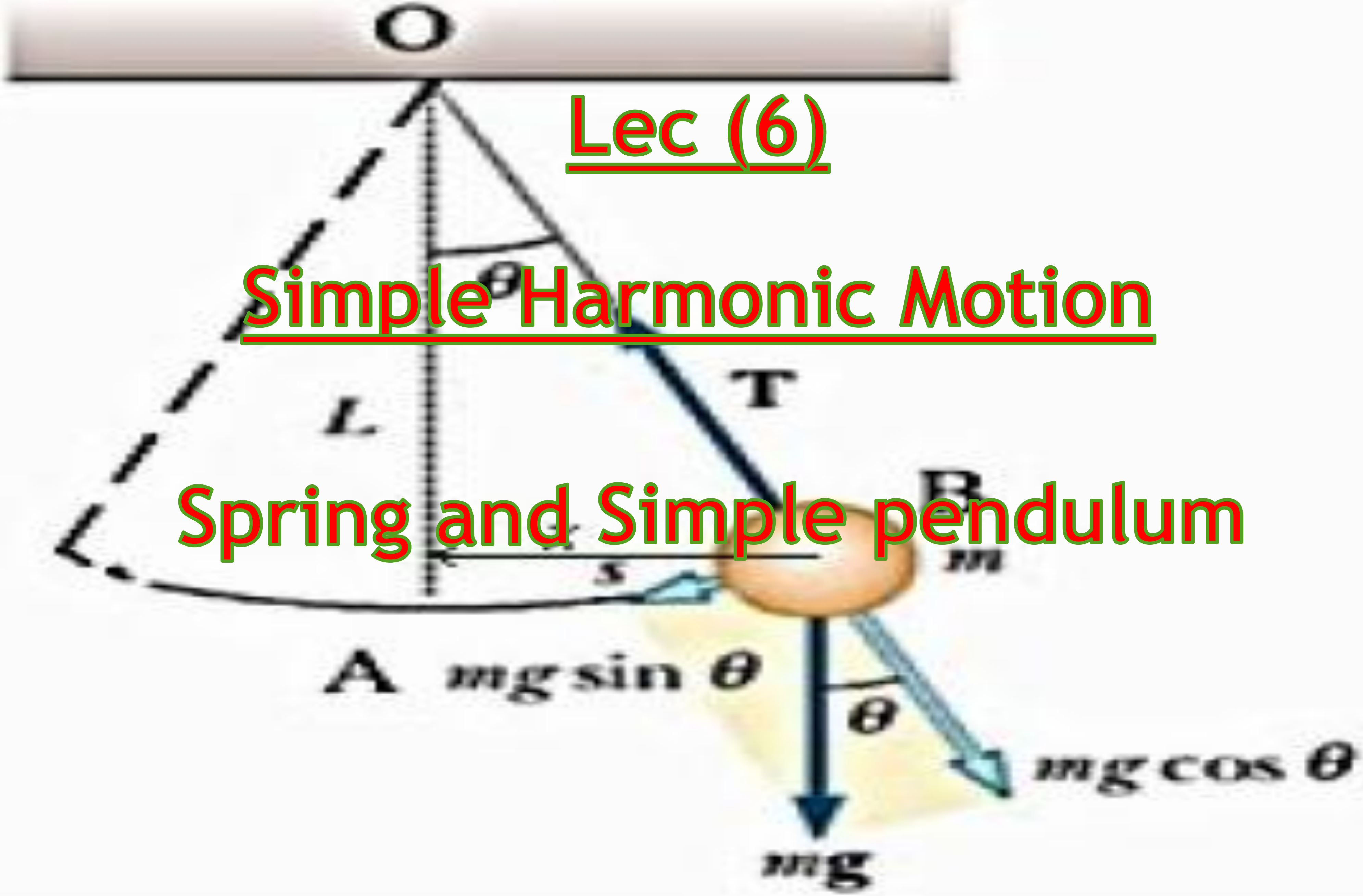
Lec (6)

Simple Harmonic Motion

Spring and Simple pendulum



Simple Harmonic Motion

- ✓ **The two and fro periodic motion is called vibration or oscillation**
- ✓ **In vibration or oscillation there is always an equilibrium position**
- ✓ **The displacement of the vibrating or oscillating particle is measured from the equilibrium position.**
- ✓ **If the acceleration of the vibrating or oscillating particle hence the force acting on it is proportional to the displacement and always directed towards equilibrium position then the motion is called 'SIMPLE HARMONIC MOTION (SHM)'.**
- ✓ **All SHM s are periodic but all periodic motions are not SHM.**

SHM

If

$F \Rightarrow$ force of the oscillating particle

$x \Rightarrow$ displacement from the equilibrium or mean position

The motion will be SHM if

$$F \propto -x$$
$$\Rightarrow F = -kx$$

Where,

$$k$$

$\Rightarrow$ Force constant of spring constant (in case of vibration of spring)

Mean or Equilibrium position: The point where the oscillating particle will remain in equilibrium.

Displacement: The distance covered by the oscillating particle from its mean position at any given instant is known as is displacement at that instant (x).

Amplitude: The maximum distance covered by an oscillating particle on either sides of its mean position is called its amplitude (A)

Hence A= maximum x

Time period: Time taken for one complete cycle of its oscillation is known as its time period (T) of its oscillation.

Frequency: Total number of complete oscillation in unit time (1s) interval is called its frequency (f) of oscillation.
The relation between the time period and frequency is

$$T = \frac{1}{f}$$

Phase: The argument, of the trigonometric function that represent a SHM, represents the state of the motion i.e., its position, velocity, acceleration etc. is called its phase.

The SHM auxiliary circle

An imaginary circular motion gives a mathematical insight into SHM. Its angular velocity is ω.

Physlets animation 'The connection between SHM and circular motion'

The time period of the motion, $T = \dfrac{2\pi}{\omega}$.

The frequency of the motion, $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$.

Displacement of the SHM, $s = A\cos(\omega t)$

SHM equations

Displacement $s = A\cos(\omega t)$

At $t = 0$, the object is released from its max displacement.

The velocity, v, (rate of change of displacement) is then given by:

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = -\omega A \sin(\omega t)$$

The acceleration, a, (rate of change of velocity) is:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -\omega^2 A\cos(\omega t)$$

Note that $a = -\omega^2 s = -(2\pi f)^2 s$

Other symbols, relationships

Displacement, $s = A\sin(\omega t)$

At t =0, the object passes through its equilibrium position.

$$v = \frac{d}{dt}[A\sin(\omega t)] = \omega A\cos(\omega t)$$

$$a = \frac{d}{dt}[\omega A\cos(\omega t)] = -\omega^2 A\sin(\omega t)$$

Phase relationship, ϕ $s = A\sin(\omega t + \phi)$

Maximum values of quantities

Simple harmonic motion:

maximum displacement = A

maximum velocity = $A\omega = 2\pi f A$

maximum acceleration = $A\omega^2 = (2\pi f)^2 A$

Circular motion:

displacement = r

velocity = $r\omega$

acceleration = $r\omega^2$

Two particular systems

Mass-on-spring,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

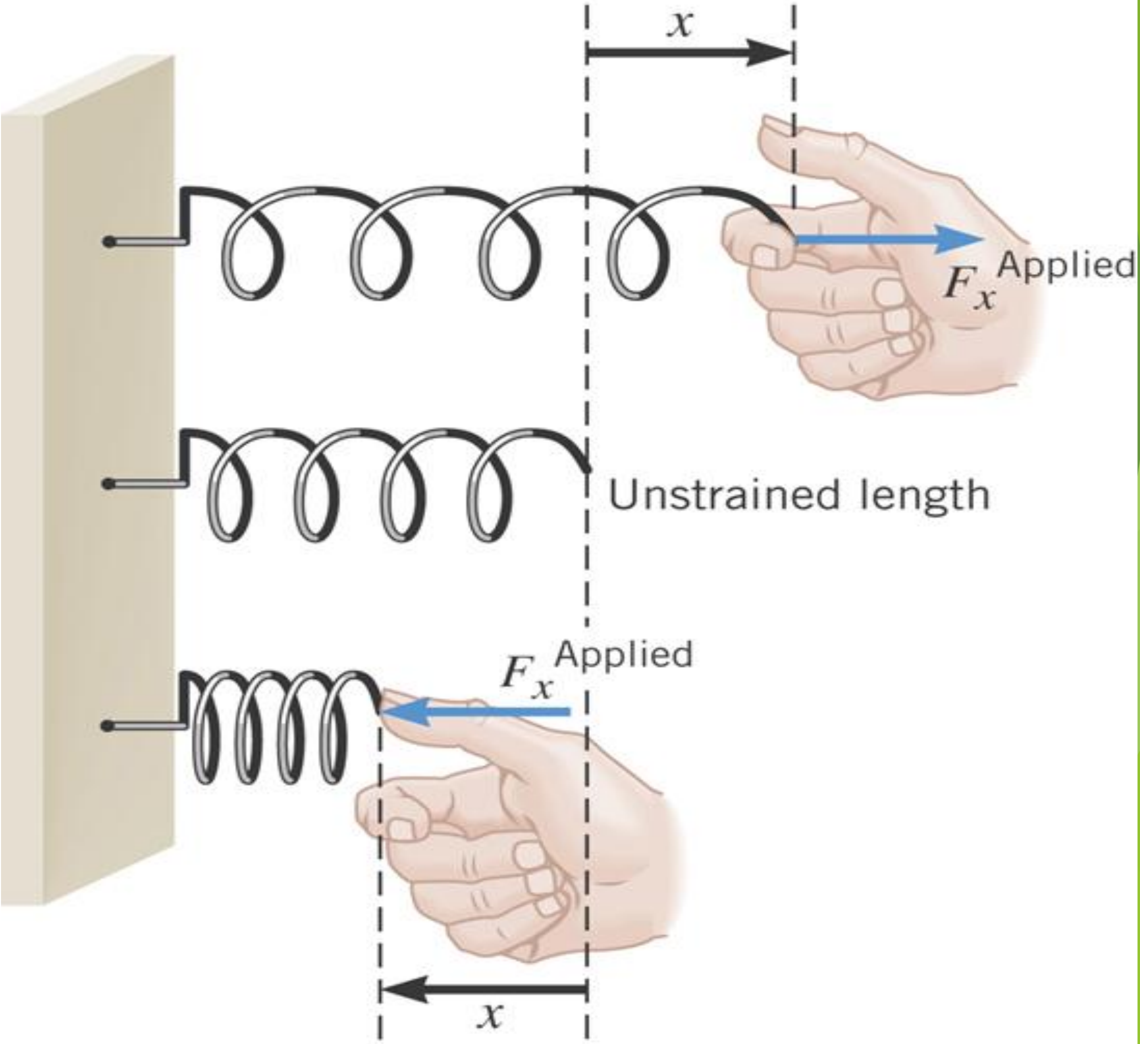
(for small angle oscillations)

The Ideal Spring and Simple Harmonic Motion

$$F_x^{Applied} = k x$$

spring constant

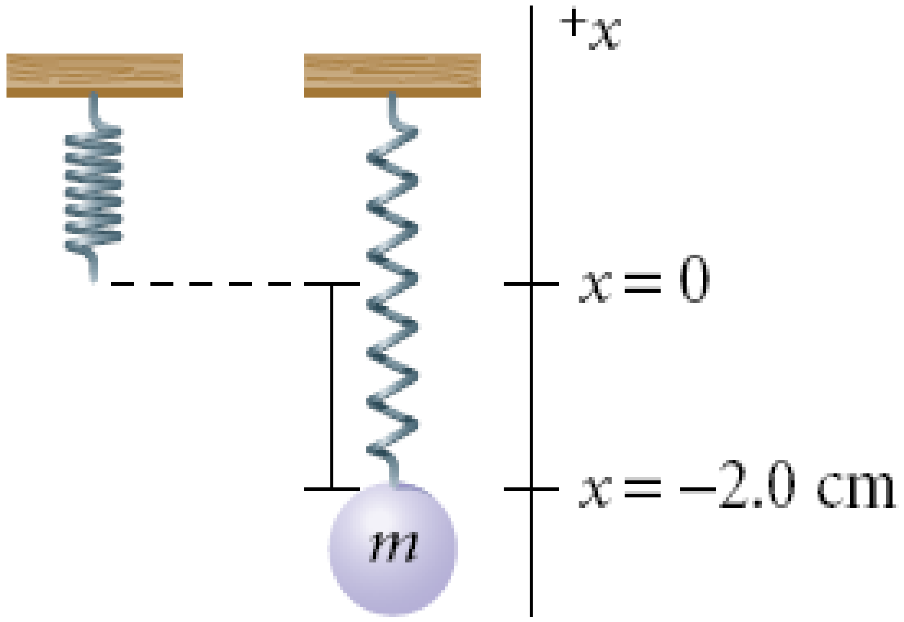
Units: N/m



Sample Problem

Hooke's Law

If a mass of 0.55 kg attached to a vertical spring stretches the spring 2.0 cm from its original equilibrium position, what is the spring constant?



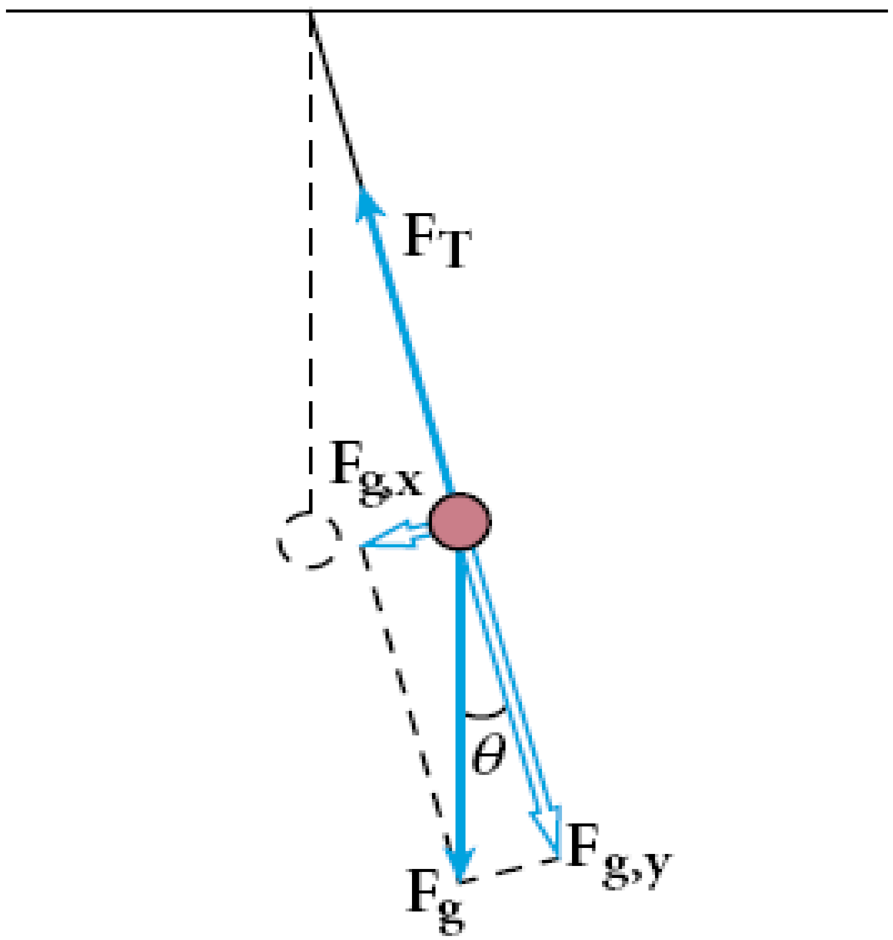
$$k = -\frac{mg}{x} = -\frac{(0.55\ \text{kg})(9.81\ \text{m/s}^2)}{-0.020\ \text{m}}$$

$$k = 270\ \text{N/m}$$

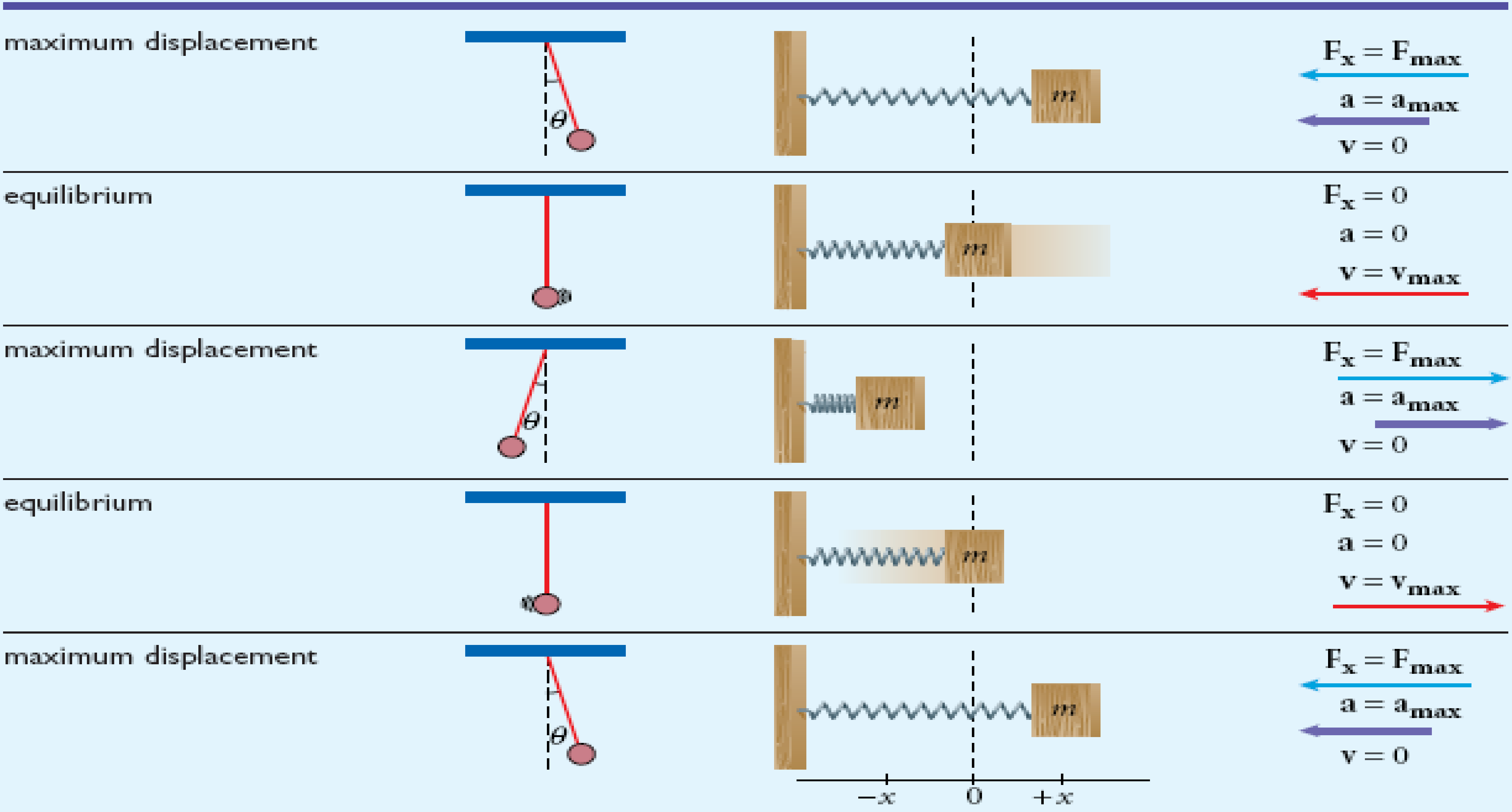
The Simple Pendulum

- A **simple pendulum** consists of a mass called a bob, which is attached to a fixed string.

The forces acting on the bob at any point are the force exerted by the string and the gravitational force.



Simple Harmonic Motion



Period of a Simple Pendulum

- The **period** of a **simple pendulum** depends on the **length** and on the **free-fall acceleration.**

$$T = 2\pi\sqrt{\frac{L}{a_g}}$$

$$\text{period} = 2\pi\sqrt{\frac{\text{length}}{\text{free-fall acceleration}}}$$

- The period does not depend on the mass of the bob or on the amplitude (for small angles).

Period of a Mass-Spring System

- The **period** of an ideal **mass-spring system** depends on the **mass** and on the **spring constant**.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{period} = 2\pi\sqrt{\frac{\text{mass}}{\text{spring constant}}}$$

- The period does not depend on the amplitude.
- This equation applies only for systems in which the spring obeys Hooke’s law.

SOLVED PROBLEM

A 100-g object is suspended from a spring whose force constant is 50 N/m. (*a*) By how much does the spring stretch? (*b*) What is the period of oscillation of the system? (*c*) What is itsfrequency?

(*a*) Here $F = mg = (0.1\ \text{kg})(9.8\ \text{m/s}^2) = 0.98\ \text{N}$, and so the spring stretches by

$$s = \frac{F}{k} = \frac{0.98\ \text{N}}{50\ \text{N/m}} = 0.0196\ \text{m} = 1.96\ \text{cm}$$

(*b*)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1\ \text{kg}}{50\ \text{N/m}}} = 0.281\ \text{s}$$

(*c*)

$$f = \frac{1}{T} = \frac{1}{0.281\ \text{s}} = 3.56\ \text{s}^{-1} = 3.56\ \text{Hz}$$

SOLVED PROBLEM

Find the length in meters of a simple pendulum whose period is 2 s.

The first step is to solve the formula $T = 2\pi\sqrt{L/g}$ for L. We proceed as follows:

$$T^2 = \frac{4\pi^2 L}{g}$$

$$L = \frac{gT^2}{4\pi^2}$$

Now we substitute $g = 9.8\ \mathrm{m/s^2}$ and $T = 2\ \mathrm{s}$, and we obtain

$$L = \frac{(9.8\ \mathrm{m/s^2})(2\ \mathrm{s})^2}{4\pi^2} = 0.993\ \mathrm{m}$$